[This question paper contains 28 printed pages.]

Your Roll No..............

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Sr. No. of Question Paper : 1758

Unique Paper Code : 12271403

Name of the Paper : Introductory Econometrics

Name of the Course : **CBCS Core**

Semester : IV

Duration : 3 Hour | Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Answer any **Five** questions out of **Seven**.
3. **All** questions carry equal marks.
4. Use of simple non-programmable calculator is allowed. Statistical tables are attached for your reference.
5. Answers may be written either in English or Hindi; but the same medium should be used throughout the paper.

छात्रों के लिए निर्देश

1. इस प्रश्न-पत्र के मिलते ही ऊपर दिए गए निर्धारित स्थान पर अपना अनुक्रमांक लिखिए ।
2. सात में से किन्हीं पाँच प्रश्नों के उत्तर दीजिए ।
3. सभी प्रश्नों के अंक समान हैं ।
4. साधारण गैर-प्रोग्राम योग्य कैलकुलेटर के उपयोग की अनुमति है । आपके संदर्भ के लिए सांख्यिकीय सारणियां संलग्न हैं ।

   सभी गणनाओं के लिए दो दशमलव स्थानों पर संख्याओं को गोल किया जा सकता है ।
5. इस प्रश्न-पत्र का उत्तर अंग्रेजी या हिंदी किसी एक भाषा में दीजिए, लेकिन सभी उत्तरों का माध्यम एक ही होना चाहिए ।

1. State whether the following statements are True or False. Give reasons for your answer.

(i) The assumptions made by the classical linear regression model (CLRM) are not necessary to compute OLS estimators.

(ii) In the model $Y_i = \beta_1 + \beta_2 X_2 + \beta_3 X_3 + \varepsilon_i$, if X_2 and X_3 are negatively correlated in the sample and $\beta_3 > 0$, omitting X_3 from the model will bias b_{12} downward [i.e., $E(b_{12}) < \beta_2$] where b_{12} is the slope coefficient in the regression of Y on X_2 alone.

(iii) In cases of high multicollinearity, it is not possible to assess the individual significance of one or more partial regression coefficients.

(iv) In the presence of heteroscedasticity, the usual OLS method always overestimates the standard errors of estimators.

(v) The Durbin-Watson d test assumes that the variance of the error term ε_t is homoscedastic. (3×5=15)

बताइए कि निम्नलिखित कथन सही हैं या गलत। अपने उत्तर के कारण बताएं।

(i) शास्त्रीय रेखीय प्रतिगमन मॉडल (CLRM) द्वारा बनाई गई धारणाएँ OLS अनुमानकों की गणना करने के लिए आवश्यक नहीं हैं।

(ii) मॉडल $Y_i = \beta_1 + \beta_2 X_2 + \beta_3 X_3 + \varepsilon_i$, if X_2 में, यदि X_2 और X_3 नमूने में नकारात्मक रूप से सहसंबंधित हैं और $\beta_3 > 0$, मॉडल से X_3 को छोड़ने से b_{12} नीचे की ओर बायस होगा [यानी, $E(b_{12}) < \beta_2$] जहां b_{12} केवल X_2 पर Y के प्रतिगमन में ढलान गुणांक हैं।

(iii) उच्च बहुसंरेखता के मामलों में, एक या अधिक आंशिक प्रतिगमन गुणांक के व्यक्तिगत महत्व का आकलन करना संभव नहीं है।

(iv) विषमलैंगिकता की उपस्थिति में, सामान्य ओएलएस विधि हमेशा अनुमानकों की मानक त्रुटियों को अधिक अनुमानित करती है।

(v) डर्बिन-वाटसन डी परीक्षण मानता है कि त्रुटि पद ε_t का प्रसरण समलिंगी है।

2. From the annual data for the cement sector for 1990-2022 the following regression results were obtained for the

$$\text{PRF: } lnY = \beta_1 + \beta_2 ln\, K + \beta_3 \ln L + \beta_4 t + \varepsilon_t$$

Model 1 :

$$\widehat{\ln Y} = 2.81 + 0.53 \ln K + 0.61 \ln L + 0.047\, t$$

se (1.38) (0.44) (0.14) (0.006)

RSS = 332.1

Model 2 :

$$\ln\widehat{(Y/L)} = -0.11 + 0.18 \ln (K/L) + 0.006t$$

(0.03) (0.05) (0.006)

RSS = 362.04 p value of F test = 0.000 $d = 0.86$

Y = index of real output, K = index of real capital input, L = index of real labour input, t = time or trend.

(i) Interpret the coefficient of In (K/L) in model 2.

(ii) What additional information will you need to conduct a goodness of fit test for model 1 and how will you use it?

(iii) Test that $\beta_2 + \beta_3 = 1$ i.e the sector faces constant returns to scale.

(iv) Can we infer that model 1 is better than model 2 as it has a higher R^2? Why?

(v) Test at 1% level of significance if model 2 suffers from first order autocorrelation. Which test will you use to check if this model suffers from second order autocorrelation? (3×5=15)

1990 - 2022 के लिए सीमेंट क्षेत्र के वार्षिक आंकड़ों से पीआरएफ $lnY = \beta_1 + \beta_2 ln\, K + \beta_3 \ln L + \beta_4 t + \varepsilon_t$ के लिए निम्नलिखित प्रतिगमन परिणाम प्राप्त हुए :

Model 1 : $\widehat{\ln Y} = 2.81 + 0.53 \ln K + 0.61 \ln L + 0.047\, t$

se (1.38) (0.44) (0.14) (0.006)

RSS = 332.1

Model 2 : $\ln\widehat{(Y/L)} = -0.11 + 0.18 \ln (K/L) + 0.006t$

(0.03) (0.05) (0.006)

RSS = 362.04 p value of F test = 0.000 $d = 0.86$

Y = वास्तविक उत्पादन का सूचकांक, K = वास्तविक पूंजी इनपुट का सूचकांक, L = वास्तविक श्रम इनपुट का सूचकांक, t = समय या प्रवृत्ति।

(i) मॉडल 2 में ln (K/L) के गुणांक की व्याख्या कीजिए।

(ii) मॉडल के लिए उपयुक्तता परीक्षण करने के लिए आपको कौन सी अतिरिक्त जानकारी की आवश्यकता होगी और आप इसका उपयोग कैसे करेंगे?

(iii) परीक्षण कीजिए कि $\beta_2 + \beta_3 = 1$ यानी क्षेत्र पैमाने पर निरंतर रिटर्न का सामना करता है।

(iv) क्या हम यह अनुमान लगा सकते हैं कि मॉडल 1 मॉडल 2 से बेहतर है क्योंकि इसमें R^2 अधिक है? क्यों?

(v) यदि मॉडल 2 प्रथम कोटि के स्वसहसंबंध से ग्रस्त है, तो 1% सार्थकता स्तर पर परीक्षण कीजिए। यह जाँचने के लिए आप किस परीक्षण का प्रयोग करेंगे कि क्या यह मॉडल दूसरे क्रम के स्वत: सहसंबंध से ग्रस्त है?

3. Two researchers are investigating the effects of time spent studying on the examination marks earned by students on a certain course. For a sample of 100 students, they have the examination mark, M, total hours spent studying. H, hours on primary study. P, and hours spent on revision, R. By definition. H = P + R. The sample means of H, P, and R are 100 hours, 95 hours, and 5 hours, respectively and the standard deviations of the distributions of H, P, and R are 10.1, 10.1. and 2.1. respectively. Mean examination marks earned by

students are 25. Researcher A decides to regress M on P and R and fits the following regression :

Regression A: $M = 45.6 + 0.15\ P + 0.21\ R$

$Se = (2.8)\quad (0.03)\qquad (0.14)$

Researcher B decides to regress M on H and P, with regression output.

Regression B: $M = 45.6 + 0.21\ H - 0.06\ P.$

$Se = (3.4)\quad (0.07)\qquad (0.04)$

(i) Give an interpretation of the coefficients of regression A.

(ii) Perform t tests of the significance of the coefficients of P and R in regression A.

(iii) The researcher says that the insignificant coefficient of R is to be expected because the students, on average, spent much less time on revision than on primary study. Explain whether this assertion is correct.

(iv) Calculate elasticities of examination marks with respect to hours of primary study for both the regressions.

(v) Do you think there is a problem with both the regressions? Why/why not? $(3\times5=15)$

दो शोधकर्ता एक निश्चित पाठ्यक्रम पर छात्रों द्वारा अर्जित परीक्षा के अंकों पर अध्ययन करने में लगने वाले समय के प्रभावों की जांच कर रहे हैं। 100 छात्रों के एक नमूने के लिए, उनके पास परीक्षा चिहन, एम, अध्ययन में बिताए कुल घंटे, एच, प्राथमिक अध्ययन पर घंटे, पी, और संशोधन पर बिताए घंटे, आर हैं। परिभाषा के अनुसार, $H = P + R$. H, P, और R का नमूना मतलब क्रमशः 100 घंटे, 95 घंटे और 5 घंटे हैं और H, P, और R के वितरण के मानक विचलन क्रमशः 10.1, 10.1 और 2.1 हैं। छात्रों द्वारा अर्जित औसत परीक्षा अंक 25 हैं।

शोधकर्ता A ने पी और आर पर एम को प्रतिगमन का फैसला किया और निम्नलिखित प्रतिगमन को फिट किया :

Regression A: $M = 45.6 + 0.15\ P + 0.21\ R$

$Se = (2.8)\quad (0.03)\quad (0.14)$

शोधकर्ता B ने प्रतिगमन आउटपुट के साथ एच और पी पर एम को प्रतिगमन का फैसला किया।

Regression B: $M = 45.6 + 0.21\ H - 0.06\ P.$

$Se = (3.4)\quad (0.07)\quad (0.04)$

(i) समाश्रयण A के गुणांकों की व्याख्या कीजिए।

(ii) प्रतिगमन A में P और R के गुणांकों के महत्त्व का t परीक्षण कीजिए।

(iii) शोधकर्ता का कहना है कि आर के नगण्य गुणांक की उम्मीद की जानी चाहिए क्योंकि छात्रों ने प्राथमिक अध्ययन की तुलना में संशोधन पर औसतन बहुत कम समय बिताया। बताएं कि क्या यह दावा सही है।

(iv) दोनों प्रतिगमन के लिए प्राथमिक अध्ययन के घंटों के संबंध में परीक्षा के अंकों की लोच की गणना कीजिए।

(v) क्या आपको लगता है कि दोनों प्रतिगमन में कोई समस्या है? क्यों, क्यों नहीं?

4. **Dependent Variable-** Y: Life Expectancy

Explanatory Variables-

X_2: Number of Under Five Deaths,

X_3: Polio Immunization Coverage,

X_4: Per capita Govt. Expenditure on Health Care,

X_5: Per Capita GNI (in Rs.),

X_6: Dummy for Democracy = 1 if the country is a democracy and 0 otherwise

No. of observations: n = 180 countries for the year 2017

$$\text{MODEL 1: } Y_i = -23.659 + .982X_2 - 0.657X_3 + .000472X_4 + .00033X_5 + 782X_6$$

$$\text{MODEL 2: } Y_i = -99.02 + .445X_4$$

$$\text{Se} \quad (42.33) \quad (0.127)$$

(i) How will you test the hypothesis that the impact of an additional Rupee of Per capita Govt. Expenditure on Health Care is same as the impact of an additional Rupee of Per Capita GNI on Life expectancy in MODEL 1?

(ii) Show that for MODEL 2 the coefficient of X_4 is an unbiased estimator of the true impact of X_4 on Y.

(iii) What are the consequences on the quality of OLS estimates if being a Democracy or not is an irrelevant variable?

(iv) Which of the parts (i) and (ii) above will you not be able to answer if it was known that the errors are not normally distributed for both MODEL 1 and MODEL 2?

(v) How will you construct a 95% confidence interval for the mean predicted life expectancy? What determines the width of this interval? (3×5=15)

आश्रित चर- Y: जीवन प्रत्याशा

विवरणात्मक परिवर्ती-

X_2: पांच से कम उम्र की मौतों की संख्या,

X_3: पोलियो प्रतिरक्षण कवरेज,

X_4: प्रति व्यक्ति सरकार। स्वास्थ्य देखभाल पर व्यय,

X_5: प्रति व्यक्ति जीएनआई (रुपये में),

X_6: डमी फॉर डेमोक्रेसी = 1 अगर देश में लोकतंत्र है और 0 अन्यथा

टिप्पणियों की संख्या: वर्ष 2017 के लिए

n = 180 देश

नमूना 1: $Y_i = -23.659 + .982X_2 - 0.657X_3 + .000472X_4 + .00033X_5 + 782X_6$

नमूना 2 : $Y_i = -99.02 + .445X_4$

Se (42.33) (0.127)

(i) आप इस परिकल्पना का परीक्षण कैसे करेंगे कि प्रति व्यक्ति सरकार के एक अतिरिक्त रुपये का प्रभाव स्वास्थ्य देखभाल पर व्यय मॉडल 1 में जीवन प्रत्याशा पर प्रति व्यक्ति GNI के अतिरिक्त रुपये के प्रभाव के समान है?

(ii) दिखाएं कि मॉडल 2 के लिए X_4 का गुणांक Y पर X_4 के वास्तविक प्रभाव का एक निष्पक्ष अनुमानक है।

(iii) लोकतंत्र होना या न होना एक अप्रासंगिक चर है तब ओएलएस अनुमानों की गुणवत्ता पर क्या परिणाम होते हैं?

(iv) ऊपर दिए गए (i) और (ii) में से कौन सा भाग आप उत्तर देने में सक्षम नहीं होंगे यदि यह ज्ञात था कि त्रुटियाँ सामान्य रूप से मॉडल 1 और मॉडल 2 दोनों के लिए वितरित नहीं की जाती हैं?

(v) आप जीवन प्रत्याशा के अनुमानित माध्य के लिए 95% विश्वास्यता अंतराल की रचना कैसे करेंगे? इस अंतराल की चौड़ाई क्या निर्धारित करती है?

5. The following model was estimated for United States from 1998 to 2021:

$$\hat{Y}_t = 10.078 - 10.337D_t - 17.549\left(\frac{1}{X_t}\right) + 38.173D_t\left(\frac{1}{X_t}\right)$$

se = (1.4204) (1.6859) (8.3373) (9.399)

$R^2 = 0.8787$ d=0.64

where Y = year-to-year percentage change in the index of hourly earnings

X = percent unemployment rate

D = 1 for 1998-2005

= 0 if otherwise

(i) What are the estimated equations of the Phillips curve for two periods 1998-2005 & 2006-2021?

(ii) Are differential intercept and slope coefficients individually statistically significant?

(iii) Suppose the error component of the model is distributed as an AR(1) scheme. How would you test for the presence of serial correlation in the model?

(iv) One of the researchers thinks that the following model would be appropriate for the relation between Y and X.

$$\widehat{Y_t} = \beta_1 + \beta_2 D_t + \beta_3 \left(\frac{1}{X_t}\right) + \beta_4 D_t \left(\frac{1}{X_t}\right) + \beta_5 Y_{t-1} + \varepsilon_t$$

How can a test for the presence of serial correlation be performed for this model?

(v) If the regression function is given as

$$\widehat{Y_t} = \beta_1 + \beta_2 \left(\frac{1}{X_t}\right) + \varepsilon_t$$

and tests have concluded that ε_t suffers from AR(1). How would you use Generalized Least Squares (GLS) to correct for the problem of autocorrelation? (3×5=15)

1998 से 2021 तक संयुक्त राज्य अमेरिका के लिए निम्नलिखित मॉडल का अनुमान लगाया गया था :

$$\widehat{Y_t} = 10.078 - 10.337D_t - 17.549\left(\frac{1}{X_t}\right) + 38.173D_t\left(\frac{1}{X_t}\right)$$

se = (1.4204) (1.6859) (8.3373) (9.399)

$R^2 = 0.8787$ d=0.64

जहाँ Y = प्रति घंटा आय के सूचकांक में वर्ष-दर-वर्ष प्रतिशत परिवर्तन

एक्स = प्रतिशत बेरोजगारी दर

1998-2005 के लिए डी = 1

= 0 यदि अन्यथा

(i) 1998 - 2005 और 2006 - 2021 की दो अवधियों के लिए फिलिप्स वक्र के अनुमानित समीकरण क्या हैं?

(ii) क्या अंतर अवरोधन और ढलान गुणांक व्यक्तिगत रूप से सांख्यिकीय रूप से महत्वपूर्ण हैं?

(iii) मान लीजिए कि मॉडल के त्रुटि घटक को AR(1) योजना के रूप में वितरित किया गया है। आप मॉडल में क्रमिक सहसंबंध की उपस्थिति का परीक्षण कैसे करेंगे?

(iv) एक शोधकर्ता सोचता है कि निम्नलिखित मॉडल Y और X के बीच संबंध के लिए उपयुक्त होगा

$$\widehat{Y_t} = \beta_1 + \beta_2 D_t + \beta_3\left(\frac{1}{X_t}\right) + \beta_4 D_t\left(\frac{1}{X_t}\right) + \beta_5 Y_{t-1} + \varepsilon_t$$

इस मॉडल के लिए सीरियल सहसंबंध की उपस्थिति के लिए परीक्षण कैसे किया जा सकता है।

(v) यदि समाश्रयण फलन इस रूप में दिया गया हो

$$\widehat{Y_t} = \beta_1 + \beta_2\left(\frac{1}{X_t}\right) + \varepsilon_t$$

और परीक्षणों ने निष्कर्ष निकाला है कि ε_t AR(1) से पीड़ित है। स्वतःसहसंबंध की समस्या को ठीक करने के लिए आप सामान्यीकृत न्यूनतम वर्ग (GLS) का उपयोग कैसे करेंगे?

6. The wage rate per month (in \$) for 50 employees (25 males and 25 females) was regressed on education (in years), experience (in years), Age (in years) and a dummy variable which takes value 1 for males and 0 otherwise.

$$\text{WAGE} = \underset{(383)}{648} + \underset{(31.7)}{132}*\text{EDUC} + \underset{(13.0)}{38.0}*\text{EXPER} - \underset{(7.69)}{5.83}*\text{AGE} + \underset{(147)}{488}*\text{DUMMY\_MALE} + e_i \qquad R^2 = 0.457$$

(standard errors in parentheses)

(i) Test for equality of variances of male and female wages if the sample variance of wages for males is 23,589 $\$^2$ and females is 19,382 $\$^2$.

(ii) Do wages of male and female employees differ statistically?

(iii) Describe the specification that can be used to model that impact of education on wages is sensitive to gender? How will you test the statistical significance of this sensitivity?

(iv) Describe a specification that can be used to model a conjecture that increase in age from 25 to 26 years will have more impact on wages than if the age increased from 50 to 51. How will you test this conjecture?

(v) How will the estimates of the regression coefficients change if another researcher using the same data takes a dummy for females DUMMY_ FEM which is 1 for females and 0 otherwise? (3×5=15)

50 कर्मचारियों (25 पुरुषों और 25 महिलाओं) के लिए प्रति माह मजदूरी दर (डॉलर में) शिक्षा (वर्षों में), अनुभव (वर्षों में), आयु (वर्षों में) और एक डमी चर जो पुरुषों के लिए मान 1 लेता है और 0 अन्यथा।

$$WAGE = \underset{(383)}{648} + \underset{(31.7)}{132}*EDUC + \underset{(13.0)}{38.0}*EXPER - \underset{(7.69)}{5.83}*AGE + \underset{(147)}{488}*DUMMY\_MALE + e_i \qquad R^2 = 0.457$$

(कोष्टक में मानक त्रुटियां)

(i) पुरुषों और महिलाओं के वेतन के अंतर की समानता के लिए परीक्षण कीजिए यदि पुरुषों के लिए मजदूरी का नमूना अंतर 23,589 $\$^2$ और महिलाओं के लिए 19,382 $\$^2$ है।

(ii) क्या पुरुष और महिला कर्मचारियों के वेतन सांख्यिकीय रूप से भिन्न हैं?

(iii) उस विनिर्देश का वर्णन कीजिए जिसका उपयोग मॉडल के लिए किया जा सकता है कि वेतन पर शिक्षा का प्रभाव लिंग के प्रति संवेदनशील है? आप इस संवेदनशीलता के सांख्यिकीय महत्व का परीक्षण कैसे करेंगे?

(iv) एक विनिर्देश का वर्णन करें जिसका उपयोग अनुमान लगाने के लिए किया जा सकता है कि 25 से 26 वर्ष की आयु में वृद्धि का मजदूरी पर अधिक प्रभाव पड़ेगा यदि आयु 50 से 51 तक बढ़ जाती है। आप इस अनुमान का परीक्षण कैसे करेंगे?

(v) यदि समान डेटा का उपयोग करने वाला एक अन्य शोधकर्ता महिलाओं के लिए एक डमी लेता है, जो महिलाओं के लिए DUMMY_ FEM है, जो महिलाओं के लिए 1 है और अन्यथा 0 है, तो प्रतिगमन गुणांक के अनुमान कैसे बदलेंगे?

7. Two individuals fit earnings functions relating EARNINGS (Salary earned per year) to S (number of years of school education). The first individual does it correctly and obtains the result :

$$\text{Earning } s_i = -13.93 + 2.46S_i + \varepsilon_i$$

The second individual makes a mistake and regresses S on EARNINGS, obtaining the following result :

$$S_i = 12.29 + 0.070 EARNINGS_i + w_i$$

p- value (.025) (.028)

(i) From this result the second individual derives

$$\text{EARNING } S_i = -175.57 + 14.29\ S_i + \varepsilon_i$$

Explain why this equation is different from that fitted by the first individuals.

(ii) Prove that the r^2 obtained from these two regressions will be the same.

(iii) Find the value of r^2.

(iv) An expert suggests to the first individual that by excluding experience (EX) from her regression, she will not obtain unbiased estimator of the coefficient of S. Explain with proof that the expert is correct.

(v) An expert recommends that the researchers should test for heteroscedasticity using the White's General test for a model with Earnings as the dependent variable and both S (number of years of school education) and EX (number of years of experience) as regressors. List the steps involved in conducting this test. The first individual found the p value of the White's General test to be 0.003. She also found using Park's test that the p value of the coefficient of In Si) was 0.00042. How can she use this information to run a Weighted Least Square model? (3×5=15)

आय (प्रति वर्ष अर्जित वेतन) से S (स्कूली शिक्षा के वर्षों की संख्या) से संबंधित आय कार्यों में दो व्यक्ति फिट होते हैं। पहला व्यक्ति इसे सही ढंग से करता है और परिणाम प्राप्त करता है :

$$\text{Earning } s_i = -13.93 + 2.46S_i + \varepsilon_i$$

दूसरा व्यक्ति एक गलती करता है और कमाई पर 5 को पीछे कर देता है, जिसके परिणामस्वरूप निम्नलिखित परिणाम प्राप्त होते हैं :

$$S_i = 12.29 + 0.070EARNINGS_i + w_i$$

p- value (.025) (.028)

(i) इस परिणाम से दूसरा व्यक्ति व्युत्पनन करता हैं

$$\text{EARNING } S_i = -175.57 + 14.29\ S_i + \varepsilon_i$$

व्याख्या कीजिए कि यह समीकरण पहले व्यक्ति द्वारा लगाए गए समीकरण से भिन्न क्यों है?

(ii) सिद्ध कीजिए कि इन दोनों प्रतिगमन से प्राप्त r^2 समान होंगे।

(iii) r^2 का मान ज्ञात कीजिए।

(iv) एक विशेषज्ञ पहले व्यक्ति को सुझाव देता है कि अपने प्रतिगमन से अनुभव (EX) को छोड़कर, वह S के गुणांक का निष्पक्ष अनुमानक प्राप्त नहीं करेगा। सबूत के साथ समझाएं कि विशेषज्ञ सही है।

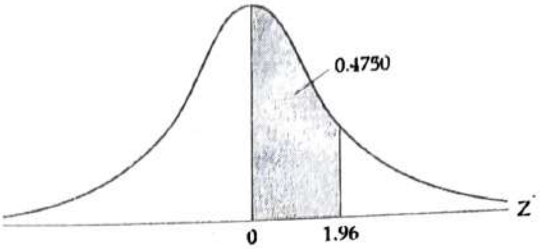
(v) एक विशेषज्ञ की सिफारिश है कि शोधकर्ताओं को एक मॉडल के लिए व्हाइट के सामान्य परीक्षण का उपयोग करते हुए विषमलैंगिकता के लिए परीक्षण करना चाहिए, जिसमें आश्रित चर के रूप में आय और दोनों S (स्कूली शिक्षा के वर्षों की संख्या) और EX (अनुभव के वर्षों की संख्या) प्रतिगामी के रूप में हों। इस परीक्षण को करने में शामिल चरणों की सूची बनाएं। पहले व्यक्ति ने व्हाइट के सामान्य परीक्षण का p मान 0.003 पाया। उसने पार्क के परीक्षण का उपयोग करते हुए यह भी पाया कि In Si के गुणांक का p मान 0.00042 था। वेटेड लीस्ट स्क्वायर (WLS) मॉडल को चलाने के लिए वह इस जानकारी का उपयोग कैसे कर सकती है?

TABLE E-1a AREAS UNDER THE STANDARDIZED NORMAL DISTRIBUTION

Example

Pr(0 ≤ Z ≤ 1.96) = 0.4750

Pr(Z ≥ 1.96) = 0.5 − 0.4750 = 0.025

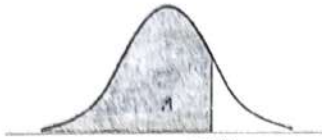


Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4454	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

Note: This table gives the area in the right-hand tail of the distribution (i.e., $Z \geq 0$). But since the normal distribution is symmetrical about $Z = 0$, the area in the left-hand tail is the same as the area in the corresponding right-hand tail. For example, $P(-1.96 \leq Z \leq 0) = 0.4750$. Therefore, $P(-1.96 \leq Z \leq 1.96) = 2(0.4750) = 0.95$.

TABLE E-1b CUMULATIVE PROBABILITIES OF THE STANDARD NORMAL DISTRIBUTION

Entry is area A under the standard normal curve from $-\infty$ to $Z(A)$



z(A)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Selected Percentiles

Cumulative probability A:	.90	.95	.975	.98	.99	.995	.999
$Z(A)$:	1.282	1.645	1.960	2.054	2.326	2.576	3.090

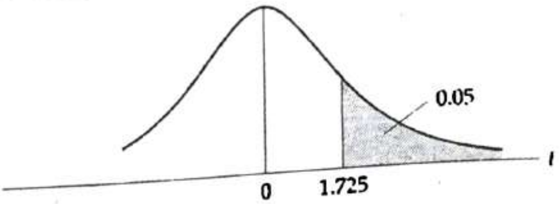
TABLE E-2 PERCENTAGE POINTS OF THE *t* DISTRIBUTION

Example

$\Pr(t > 2.086) = 0.025$

$\Pr(t > 1.725) = 0.05$ for d.f. = 20

$\Pr(|t| > 1.725) = 0.10$



d.f. \ Pr	0.25 0.50	0.10 0.20	0.05 0.10	0.025 0.05	0.01 0.02	0.005 0.010	0.001 0.002
1	1.000	3.078	6.314	12.706	31.821	63.657	318.31
2	0.816	1.886	2.920	4.303	6.965	9.925	22.327
3	0.765	1.638	2.353	3.182	4.541	5.841	10.214
4	0.741	1.533	2.132	2.776	3.747	4.604	7.173
5	0.727	1.476	2.015	2.571	3.365	4.032	5.893
6	0.718	1.440	1.943	2.447	3.143	3.707	5.208
7	0.711	1.415	1.895	2.365	2.998	3.499	4.785
8	0.706	1.397	1.860	2.306	2.896	3.355	4.501
9	0.703	1.383	1.833	2.262	2.821	3.250	4.297
10	0.700	1.372	1.812	2.228	2.764	3.169	4.144
11	0.697	1.363	1.796	2.201	2.718	3.106	4.025
12	0.695	1.356	1.782	2.179	2.681	3.055	3.930
13	0.694	1.350	1.771	2.160	2.650	3.012	3.852
14	0.692	1.345	1.761	2.145	2.624	2.977	3.787
15	0.691	1.341	1.753	2.131	2.602	2.947	3.733
16	0.690	1.337	1.746	2.120	2.583	2.921	3.686
17	0.689	1.333	1.740	2.110	2.567	2.898	3.646
18	0.688	1.330	1.734	2.101	2.552	2.878	3.610
19	0.688	1.328	1.729	2.093	2.539	2.861	3.579
20	0.687	1.325	1.725	2.086	2.528	2.845	3.552
21	0.686	1.323	1.721	2.080	2.518	2.831	3.527
22	0.686	1.321	1.717	2.074	2.508	2.819	3.505
23	0.685	1.319	1.714	2.069	2.500	2.807	3.485
24	0.685	1.318	1.711	2.064	2.492	2.797	3.467
25	0.684	1.316	1.708	2.060	2.485	2.787	3.450
26	0.684	1.315	1.706	2.056	2.479	2.779	3.435
27	0.684	1.314	1.703	2.052	2.473	2.771	3.421
28	0.683	1.313	1.701	2.048	2.467	2.763	3.408
29	0.683	1.311	1.699	2.045	2.462	2.756	3.396
30	0.683	1.310	1.697	2.042	2.457	2.750	3.385
40	0.681	1.303	1.684	2.021	2.423	2.704	3.307
60	0.679	1.296	1.671	2.000	2.390	2.660	3.232
120	0.677	1.289	1.658	1.980	2.358	2.617	3.160
∞	0.674	1.282	1.645	1.960	2.326	2.576	3.090

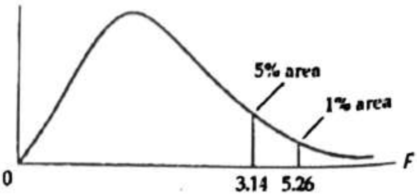
Note: The smaller probability shown at the head of each column is the area in one tail; the larger probability is the area in both tails.

Source: From E. S. Pearson and H. O. Hartley, eds., *Biometrika Tables for Statisticians,* vol. 1, 3rd ed., Table 12, Cambridge University Press, New York, 1966. Reproduced by permission of the editors and trustees of *Biometrika.*

TABLE E-3 UPPER PERCENTAGE POINTS OF THE F DISTRIBUTION

Example

$\Pr(F > 1.59) = 0.25$
$\Pr(F > 2.42) = 0.10$ for d.f. $N_1 = 10$
$\Pr(F > 3.14) = 0.05$ and $N_2 = 9$
$\Pr(F > 5.26) = 0.01$



d.f. for denominator N_2	d.f. for numerator N_1												
	Pr	1	2	3	4	5	6	7	8	9	10	11	12
1	.25	5.83	7.50	8.20	8.58	8.82	8.98	9.10	9.19	9.26	9.32	9.36	9.41
	.10	39.90	49.50	53.60	55.80	57.20	58.20	58.90	59.40	59.90	60.20	60.50	60.70
	.05	161.00	200.00	216.00	225.00	230.00	234.00	237.00	239.00	241.00	242.00	243.00	244.00
2	.25	2.57	3.00	3.15	3.23	3.28	3.31	3.34	3.35	3.37	3.38	3.39	3.39
	.10	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38	9.39	9.40	9.41
	.05	18.50	19.00	19.20	19.20	19.30	19.30	19.40	19.40	19.40	19.40	19.40	19.40
	.01	98.50	99.00	99.20	99.20	99.30	99.30	99.40	99.40	99.40	99.40	99.40	99.40
3	.25	2.02	2.28	2.36	2.39	2.41	2.42	2.43	2.44	2.44	2.44	2.45	2.45
	.10	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24	5.23	5.22	5.22
	.05	10.10	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.76	8.74
	.01	34.10	30.80	29.50	28.70	28.20	27.90	27.70	27.50	27.30	27.20	27.10	27.10
4	.25	1.81	2.00	2.05	2.06	2.07	2.08	2.08	2.08	2.08	2.08	2.08	2.08
	.10	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94	3.92	3.91	3.90
	.05	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.94	5.91
	.01	21.20	18.00	16.70	16.00	15.50	15.20	15.00	14.80	14.70	14.50	14.40	14.40
5	.25	1.69	1.85	1.88	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89
	.10	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32	3.30	3.28	3.27
	.05	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.71	4.68
	.01	16.30	13.30	12.10	11.40	11.00	10.70	10.50	10.30	10.20	10.10	9.96	9.89
6	.25	1.62	1.76	1.78	1.79	1.79	1.78	1.78	1.78	1.77	1.77	1.77	1.77
	.10	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96	2.94	2.92	2.90
	.05	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.03	4.00
	.01	13.70	10.90	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.79	7.72
7	.25	1.57	1.70	1.72	1.72	1.71	1.71	1.70	1.70	1.69	1.69	1.69	1.68
	.10	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72	2.70	2.68	2.67
	.05	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.60	3.57
	.01	12.20	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.54	6.47
8	.25	1.54	1.66	1.67	1.66	1.66	1.65	1.64	1.64	1.63	1.63	1.63	1.62
	.10	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56	2.54	2.52	2.50
	.05	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.31	3.28
	.01	11.30	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.73	5.67
9	.25	1.51	1.62	1.63	1.63	1.62	1.61	1.60	1.60	1.59	1.59	1.58	1.58
	.10	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44	2.42	2.40	2.38
	.05	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.10	3.07
	.01	10.60	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.18	5.11

Source: From E. S. Pearson and H. O. Hartley, eds., *Biometrika Tables for Statisticians*, vol. 1, 3rd ed., Table 18, Cambridge University Press, New York, 1966. Reproduced by permission of the editors and trustees of *Biometrika*.

P.T.O.

d.f. for numerator N_1													d.f. for denominator N_2
15	20	24	30	40	50	60	100	120	200	500	∞	Pr	N_2
9.49	9.58	9.63	9.67	9.71	9.74	9.76	9.78	9.80	9.82	9.84	9.85	.25	1
61.20	61.70	62.00	62.30	62.50	62.70	62.80	63.00	63.10	63.20	63.30	63.30	.10	
246.00	248.00	249.00	250.00	251.00	252.00	252.00	253.00	253.00	254.00	254.00	254.00	.05	
3.41	3.43	3.43	3.44	3.45	3.45	3.46	3.47	3.47	3.48	3.48	3.48	.25	2
9.42	9.44	9.45	9.46	9.47	9.47	9.47	9.48	9.48	9.49	9.49	9.49	.10	
19.40	19.40	19.50	19.50	19.50	19.50	19.50	19.50	19.50	19.50	19.50	19.50	.05	
99.40	99.40	99.50	99.50	99.50	99.50	99.50	99.50	99.50	99.50	99.50	99.50	.01	
2.46	2.46	2.46	2.47	2.47	2.47	2.47	2.47	2.47	2.47	2.47	2.47	.25	3
5.20	5.18	5.18	5.17	5.16	5.15	5.15	5.14	5.14	5.14	5.14	5.13	.10	
8.70	8.66	8.64	8.62	8.59	8.58	8.57	8.55	8.55	8.54	8.53	8.53	.05	
26.90	26.70	26.60	26.50	26.40	26.40	26.30	26.20	26.20	26.20	26.10	26.10	.01	
2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	.25	4
3.87	3.84	3.83	3.82	3.80	3.80	3.79	3.78	3.78	3.77	3.76	3.76	.10	
5.86	5.80	5.77	5.75	5.72	5.70	5.69	5.66	5.66	5.65	5.64	5.63	.05	
14.20	14.00	13.90	13.80	13.70	13.70	13.70	13.60	13.60	13.50	13.50	13.50	.01	
1.89	1.88	1.88	1.88	1.88	1.88	1.87	1.87	1.87	1.87	1.87	1.87	.25	5
3.24	3.21	3.19	3.17	3.16	3.15	3.14	3.13	3.12	3.12	3.11	3.10	.10	
4.62	4.56	4.53	4.50	4.46	4.44	4.43	4.41	4.40	4.39	4.37	4.36	.05	
9.72	9.55	9.47	9.38	9.29	9.24	9.20	9.13	9.11	9.08	9.04	9.02	.01	
1.76	1.76	1.75	1.75	1.75	1.75	1.74	1.74	1.74	1.74	1.74	1.74	.25	6
2.87	2.84	2.82	2.80	2.78	2.77	2.76	2.75	2.74	2.73	2.73	2.72	.10	
3.94	3.87	3.84	3.81	3.77	3.75	3.74	3.71	3.70	3.69	3.68	3.67	.05	
7.56	7.40	7.31	7.23	7.14	7.09	7.06	6.99	6.97	6.93	6.90	6.88	.01	
1.68	1.67	1.67	1.66	1.66	1.66	1.65	1.65	1.65	1.65	1.65	1.65	.25	7
2.63	2.59	2.58	2.56	2.54	2.52	2.51	2.50	2.49	2.48	2.48	2.47	.10	
3.51	3.44	3.41	3.38	3.34	3.32	3.30	3.27	3.27	3.25	3.24	3.23	.05	
6.31	6.16	6.07	5.99	5.91	5.86	5.82	5.75	5.74	5.70	5.67	5.65	.01	
1.62	1.61	1.60	1.60	1.59	1.59	1.59	1.58	1.58	1.58	1.58	1.58	.25	8
2.46	2.42	2.40	2.38	2.36	2.35	2.34	2.32	2.32	2.31	2.30	2.29	.10	
3.22	3.15	3.12	3.08	3.04	2.02	3.01	2.97	2.97	2.95	2.94	2.93	.05	
5.52	5.36	5.28	5.20	5.12	5.07	5.03	4.96	4.95	4.91	4.88	4.86	.01	
1.57	1.56	1.56	1.55	1.55	1.54	1.54	1.53	1.53	1.53	1.53	1.53	.25	9
2.34	2.30	2.28	2.25	2.23	2.22	2.21	2.19	2.18	2.17	2.17	2.16	.10	
3.01	2.94	2.90	2.86	2.83	2.80	2.79	2.76	2.75	2.73	2.72	2.71	.05	
4.96	4.81	4.73	4.65	4.57	4.52	4.48	4.42	4.40	4.36	4.33	4.31	.01	

TABLE E-3 UPPER PERCENTAGE POINTS OF THE F DISTRIBUTION (CONTINUED)

d.f. for denominator N_2		d.f. for numerator N_1											
	Pr	1	2	3	4	5	6	7	8	9	10	11	12
10	.25	1.49	1.60	1.60	1.59	1.59	1.58	1.57	1.56	1.56	1.55	1.55	1.54
	.10	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35	2.32	2.30	2.28
	.05	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.94	2.91
	.01	10.00	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.77	4.71
11	.25	1.47	1.58	1.58	1.57	1.56	1.55	1.54	1.53	1.53	1.52	1.52	1.51
	.10	3.23	2.86	2.66	2.54	2.45	2.39	2.34	2.30	2.27	2.25	2.23	2.21
	.05	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.82	2.79
	.01	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.46	4.40
12	.25	1.46	1.56	1.56	1.55	1.54	1.53	1.52	1.51	1.51	1.50	1.50	1.49
	.10	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21	2.19	2.17	2.15
	.05	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.72	2.69
	.01	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.22	4.16
13	.25	1.45	1.55	1.55	1.53	1.52	1.51	1.50	1.49	1.49	1.48	1.47	1.47
	.10	3.14	2.76	2.56	2.43	2.35	2.28	2.23	2.20	2.16	2.14	2.12	2.10
	.05	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.63	2.60
	.01	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	4.02	3.96
14	.25	1.44	1.53	1.53	1.52	1.51	1.50	1.49	1.48	1.47	1.46	1.46	1.45
	.10	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.12	2.10	2.08	2.05
	.05	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.57	2.53
	.01	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94	3.86	3.80
15	.25	1.43	1.52	1.52	1.51	1.49	1.48	1.47	1.46	1.46	1.45	1.44	1.44
	.10	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09	2.06	2.04	2.02
	.05	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.51	2.48
	.01	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.73	3.67
16	.25	1.42	1.51	1.51	1.50	1.48	1.47	1.46	1.45	1.44	1.44	1.44	1.43
	.10	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.06	2.03	2.01	1.99
	.05	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.46	2.42
	.01	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.62	3.55
17	.25	1.42	1.51	1.50	1.49	1.47	1.46	1.45	1.44	1.43	1.43	1.42	1.41
	.10	3.03	2.64	2.44	2.31	2.22	2.15	2.10	2.06	2.03	2.00	1.98	1.96
	.05	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.41	2.38
	.01	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.52	3.46
18	.25	1.41	1.50	1.49	1.48	1.46	1.45	1.44	1.43	1.42	1.42	1.41	1.40
	.10	3.01	2.62	2.42	2.29	2.20	2.13	2.08	2.04	2.00	1.98	1.96	1.93
	.05	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.37	2.34
	.01	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.43	3.37
19	.25	1.41	1.49	1.49	1.47	1.46	1.44	1.43	1.42	1.41	1.41	1.40	1.40
	.10	2.99	2.61	2.40	2.27	2.18	2.11	2.06	2.02	1.98	1.96	1.94	1.91
	.05	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.34	2.31
	.01	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.36	3.30
20	.25	1.40	1.49	1.48	1.46	1.45	1.44	1.43	1.42	1.41	1.40	1.39	1.39
	.10	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96	1.94	1.92	1.89
	.05	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.31	2.28
	.01	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.29	3.23

d.f. for numerator N_1													d.f. for denominator N_2
15	20	24	30	40	50	60	100	120	200	500	∞	Pr	
1.53	1.52	1.52	1.51	1.51	1.50	1.50	1.49	1.49	1.49	1.48	1.48	.25	
2.24	2.20	2.18	2.16	2.13	2.12	2.11	2.09	2.08	2.07	2.06	2.06	.10	10
2.85	2.77	2.74	2.70	2.66	2.64	2.62	2.59	2.58	2.56	2.55	2.54	.05	
4.56	4.41	4.33	4.25	4.17	4.12	4.08	4.01	4.00	3.96	3.93	3.91	.01	
1.50	1.49	1.49	1.48	1.47	1.47	1.47	1.46	1.46	1.46	1.45	1.45	.25	
2.17	2.12	2.10	2.08	2.05	2.04	2.03	2.00	2.00	1.99	1.98	1.97	.10	11
2.72	2.65	2.61	2.57	2.53	2.51	2.49	2.46	2.45	2.43	2.42	2.40	.05	
4.25	4.10	4.02	3.94	3.86	3.81	3.78	3.71	3.69	3.66	3.62	3.60	.01	
1.48	1.47	1.46	1.45	1.45	1.44	1.44	1.43	1.43	1.43	1.42	1.42	.25	
2.10	2.06	2.04	2.01	1.99	1.97	1.96	1.94	1.93	1.92	1.91	1.90	.10	12
2.62	2.54	2.51	2.47	2.43	2.40	2.38	2.35	2.34	2.32	2.31	2.30	.05	
4.01	3.86	3.78	3.70	3.62	3.57	3.54	3.47	3.45	3.41	3.38	3.36	.01	
1.46	1.45	1.44	1.43	1.42	1.42	1.42	1.41	1.41	1.40	1.40	1.40	.25	
2.05	2.01	1.98	1.96	1.93	1.92	1.90	1.88	1.88	1.86	1.85	1.85	.10	13
2.53	2.46	2.42	2.38	2.34	2.31	2.30	2.26	2.25	2.23	2.22	2.21	.05	
3.82	3.66	3.59	3.51	3.43	3.38	3.34	3.27	3.25	3.22	3.19	3.17	.01	
1.44	1.43	1.42	1.41	1.41	1.40	1.40	1.39	1.39	1.39	1.38	1.38	.25	
2.01	1.96	1.94	1.91	1.89	1.87	1.86	1.83	1.83	1.82	1.80	1.80	.10	14
2.46	2.39	2.35	2.31	2.27	2.24	2.22	2.19	2.18	2.16	2.14	2.13	.05	
3.66	3.51	3.43	3.35	3.27	3.22	3.18	3.11	3.09	3.06	3.03	3.00	.01	
1.43	1.41	1.41	1.40	1.39	1.39	1.38	1.38	1.37	1.37	1.36	1.36	.25	
1.97	1.92	1.90	1.87	1.85	1.83	1.82	1.79	1.79	1.77	1.76	1.76	.10	15
2.40	2.33	2.29	2.25	2.20	2.18	2.16	2.12	2.11	2.10	2.08	2.07	.05	
3.52	3.37	3.29	3.21	3.13	3.08	3.05	2.98	2.96	2.92	2.89	2.87	.01	
1.41	1.40	1.39	1.38	1.37	1.37	1.36	1.36	1.35	1.35	1.34	1.34	.25	
1.94	1.89	1.87	1.84	1.81	1.79	1.78	1.76	1.75	1.74	1.73	1.72	.10	16
2.35	2.28	2.24	2.19	2.15	2.12	2.11	2.07	2.06	2.04	2.02	2.01	.05	
3.41	3.26	3.18	3.10	3.02	2.97	2.93	2.86	2.84	2.81	2.78	2.75	.01	
1.40	1.39	1.38	1.37	1.36	1.35	1.35	1.34	1.34	1.34	1.33	1.33	.25	
1.91	1.86	1.84	1.81	1.78	1.76	1.75	1.73	1.72	1.71	1.69	1.69	.10	17
2.31	2.23	2.19	2.15	2.10	2.08	2.06	2.02	2.01	1.99	1.97	1.96	.05	
3.31	3.16	3.08	3.00	2.92	2.87	2.83	2.76	2.75	2.71	2.68	2.65	.01	
1.39	1.38	1.37	1.36	1.35	1.34	1.34	1.33	1.33	1.32	1.32	1.32	.25	
1.89	1.84	1.81	1.78	1.75	1.74	1.72	1.70	1.69	1.68	1.67	1.66	.10	18
2.27	2.19	2.15	2.11	2.06	2.04	2.02	1.98	1.97	1.95	1.93	1.92	.05	
3.23	3.08	3.00	2.92	2.84	2.78	2.75	2.68	2.66	2.62	2.59	2.57	.01	
1.38	1.37	1.36	1.35	1.34	1.33	1.33	1.32	1.32	1.31	1.31	1.30	.25	
1.86	1.81	1.79	1.76	1.73	1.71	1.70	1.67	1.67	1.65	1.64	1.63	.10	19
2.23	2.16	2.11	2.07	2.03	2.00	1.98	1.94	1.93	1.91	1.89	1.88	.05	
3.15	3.00	2.92	2.84	2.76	2.71	2.67	2.60	2.58	2.55	2.51	2.49	.01	
1.37	1.36	1.35	1.34	1.33	1.33	1.32	1.31	1.31	1.30	1.30	1.29	.25	
1.84	1.79	1.77	1.74	1.71	1.69	1.68	1.65	1.64	1.63	1.62	1.61	.10	20
2.20	2.12	2.08	2.04	1.99	1.97	1.95	1.91	1.90	1.88	1.86	1.84	.05	
3.09	2.94	2.86	2.78	2.69	2.64	2.61	2.54	2.52	2.48	2.44	2.42	.01	

TABLE E-3 UPPER PERCENTAGE POINTS OF THE F DISTRIBUTION (CONTINUED)

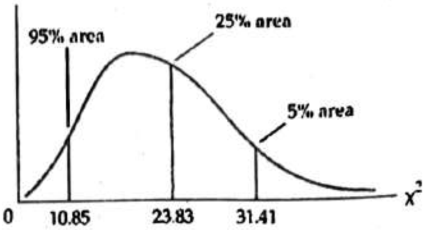
d.f. for denominator N_2	d.f. for numerator N_1												
	Pr	1	2	3	4	5	6	7	8	9	10	11	12
22	.25	1.40	1.48	1.47	1.45	1.44	1.42	1.41	1.40	1.39	1.39	1.38	1.37
	.10	2.95	2.56	2.35	2.22	2.13	2.06	2.01	1.97	1.93	1.90	1.88	1.86
	.05	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.26	2.23
	.01	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.18	3.12
24	.25	1.39	1.47	1.46	1.44	1.43	1.41	1.40	1.39	1.38	1.38	1.37	1.36
	.10	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91	1.88	1.85	1.83
	.05	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.21	2.18
	.01	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.09	3.03
26	.25	1.38	1.46	1.45	1.44	1.42	1.41	1.39	1.38	1.37	1.37	1.36	1.35
	.10	2.91	2.52	2.31	2.17	2.08	2.01	1.96	1.92	1.88	1.86	1.84	1.81
	.05	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.18	2.15
	.01	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	3.02	2.96
28	.25	1.38	1.46	1.45	1.43	1.41	1.40	1.39	1.38	1.37	1.36	1.35	1.34
	.10	2.89	2.50	2.29	2.16	2.06	2.00	1.94	1.90	1.87	1.84	1.81	1.79
	.05	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.15	2.12
	.01	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.96	2.90
30	.25	1.38	1.45	1.44	1.42	1.41	1.39	1.38	1.37	1.36	1.35	1.35	1.34
	.10	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85	1.82	1.79	1.77
	.05	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.13	2.09
	.01	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.91	2.84
40	.25	1.36	1.44	1.42	1.40	1.39	1.37	1.36	1.35	1.34	1.33	1.32	1.31
	.10	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79	1.76	1.73	1.71
	.05	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.04	2.00
	.01	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.73	2.66
60	.25	1.35	1.42	1.41	1.38	1.37	1.35	1.33	1.32	1.31	1.30	1.29	1.29
	.10	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74	1.71	1.68	1.66
	.05	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.95	1.92
	.01	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.56	2.50
120	.25	1.34	1.40	1.39	1.37	1.35	1.33	1.31	1.30	1.29	1.28	1.27	1.26
	.10	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.68	1.65	1.62	1.60
	.05	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.87	1.83
	.01	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.40	2.34
200	.25	1.33	1.39	1.38	1.36	1.34	1.32	1.31	1.29	1.28	1.27	1.26	1.25
	.10	2.73	2.33	2.11	1.97	1.88	1.80	1.75	1.70	1.66	1.63	1.60	1.57
	.05	3.89	3.04	2.65	2.42	2.26	2.14	2.06	1.98	1.93	1.88	1.84	1.80
	.01	6.76	4.71	3.88	3.41	3.11	2.89	2.73	2.60	2.50	2.41	2.34	2.27
∞	.25	1.32	1.39	1.37	1.35	1.33	1.31	1.29	1.28	1.27	1.25	1.24	1.24
	.10	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63	1.60	1.57	1.55
	.05	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.79	1.75
	.01	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.25	2.18

d.f. for numerator N_1													d.f. for denominator N_2
15	20	24	30	40	50	60	100	120	200	500	∞	Pr	
1.36	1.34	1.33	1.32	1.31	1.31	1.30	1.30	1.30	1.29	1.29	1.28	.25	22
1.81	1.76	1.73	1.70	1.67	1.65	1.64	1.61	1.60	1.59	1.58	1.57	.10	
2.15	2.07	2.03	1.98	1.94	1.91	1.89	1.85	1.84	1.82	1.80	1.78	.05	
2.98	2.83	2.75	2.67	2.58	2.53	2.50	2.42	2.40	2.36	2.33	2.31	.01	
1.35	1.33	1.32	1.31	1.30	1.29	1.29	1.28	1.28	1.27	1.27	1.26	.25	24
1.78	1.73	1.70	1.67	1.64	1.62	1.61	1.58	1.57	1.56	1.54	1.53	.10	
2.11	2.03	1.98	1.94	1.89	1.86	1.84	1.80	1.79	1.77	1.75	1.73	.05	
2.89	2.74	2.66	2.58	2.49	2.44	2.40	2.33	2.31	2.27	2.24	2.21	.01	
1.34	1.32	1.31	1.30	1.29	1.28	1.28	1.26	1.26	1.26	1.25	1.25	.25	26
1.76	1.71	1.68	1.65	1.61	1.59	1.58	1.55	1.54	1.53	1.51	1.50	.10	
2.07	1.99	1.95	1.90	1.85	1.82	1.80	1.76	1.75	1.73	1.71	1.69	.05	
2.81	2.66	2.58	2.50	2.42	2.36	2.33	2.25	2.23	2.19	2.16	2.13	.01	
1.33	1.31	1.30	1.29	1.28	1.27	1.27	1.26	1.25	1.25	1.24	1.24	.25	28
1.74	1.69	1.66	1.63	1.59	1.57	1.56	1.53	1.52	1.50	1.49	1.48	.10	
2.04	1.96	1.91	1.87	1.82	1.79	1.77	1.73	1.71	1.69	1.67	1.65	.05	
2.75	2.60	2.52	2.44	2.35	2.30	2.26	2.19	2.17	2.13	2.09	2.06	.01	
1.32	1.30	1.29	1.28	1.27	1.26	1.26	1.25	1.24	1.24	1.23	1.23	.25	30
1.72	1.67	1.64	1.61	1.57	1.55	1.54	1.51	1.50	1.48	1.47	1.46	.10	
2.01	1.93	1.89	1.84	1.79	1.76	1.74	1.70	1.68	1.66	1.64	1.62	.05	
2.70	2.55	2.47	2.39	2.30	2.25	2.21	2.13	2.11	2.07	2.03	2.01	.01	
1.30	1.28	1.26	1.25	1.24	1.23	1.22	1.21	1.21	1.20	1.19	1.19	.25	40
1.66	1.61	1.57	1.54	1.51	1.48	1.47	1.43	1.42	1.41	1.39	1.38	.10	
1.92	1.84	1.79	1.74	1.69	1.66	1.64	1.59	1.58	1.55	1.53	1.51	.05	
2.52	2.37	2.29	2.20	2.11	2.06	2.02	1.94	1.92	1.87	1.83	1.80	.01	
1.27	1.25	1.24	1.22	1.21	1.20	1.19	1.17	1.17	1.16	1.15	1.15	.25	60
1.60	1.54	1.51	1.48	1.44	1.41	1.40	1.36	1.35	1.33	1.31	1.29	.10	
1.84	1.75	1.70	1.65	1.59	1.56	1.53	1.48	1.47	1.44	1.41	1.39	.05	
2.35	2.20	2.12	2.03	1.94	1.88	1.84	1.75	1.73	1.68	1.63	1.60	.01	
1.24	1.22	1.21	1.19	1.18	1.17	1.16	1.14	1.13	1.12	1.11	1.10	.25	120
1.55	1.48	1.45	1.41	1.37	1.34	1.32	1.27	1.26	1.24	1.21	1.19	.10	
1.75	1.66	1.61	1.55	1.50	1.46	1.43	1.37	1.35	1.32	1.28	1.25	.05	
2.19	2.03	1.95	1.86	1.76	1.70	1.66	1.56	1.53	1.48	1.42	1.38	.01	
1.23	1.21	1.20	1.18	1.16	1.14	1.12	1.11	1.10	1.09	1.08	1.06	.25	200
1.52	1.46	1.42	1.38	1.34	1.31	1.28	1.24	1.22	1.20	1.17	1.14	.10	
1.72	1.62	1.57	1.52	1.46	1.41	1.39	1.32	1.29	1.26	1.22	1.19	.05	
2.13	1.97	1.89	1.79	1.69	1.63	1.58	1.48	1.44	1.39	1.33	1.28	.01	
1.22	1.19	1.18	1.16	1.14	1.13	1.12	1.09	1.08	1.07	1.04	1.00	.25	∞
1.49	1.42	1.38	1.34	1.30	1.26	1.24	1.18	1.17	1.13	1.08	1.00	.10	
1.67	1.57	1.52	1.46	1.39	1.35	1.32	1.24	1.22	1.17	1.11	1.00	.05	
2.04	1.88	1.79	1.70	1.59	1.52	1.47	1.36	1.32	1.25	1.15	1.00	.01	

TABLE E-4 UPPER PERCENTAGE POINTS OF THE χ^2 DISTRIBUTION

Example

$\Pr(\chi^2 > 10.85) = 0.95$
$\Pr(\chi^2 > 23.83) = 0.25$ for d.f. = 20
$\Pr(\chi^2 > 31.41) = 0.05$



Degrees of Freedom \ Pr	.995	.990	.975	.950	.900
1	392704×10^{-10}	157088×10^{-9}	982069×10^{-9}	393214×10^{-8}	.0158
2	.0100	.0201	.0506	.1026	.2107
3	.0717	.1148	.2158	.3518	.5844
4	.2070	.2971	.4844	.7107	1.0636
5	.4117	.5543	.8312	1.1455	1.6103
6	.6757	.8721	1.2373	1.6354	2.2041
7	.9893	1.2390	1.6899	2.1674	2.8331
8	1.3444	1.6465	2.1797	2.7326	3.4895
9	1.7349	2.0879	2.7004	3.3251	4.1682
10	2.1559	2.5582	3.2470	3.9403	4.8652
11	2.6032	3.0535	3.8158	4.5748	5.5778
12	3.0738	3.5706	4.4038	5.2260	6.3038
13	3.5650	4.1069	5.0087	5.8919	7.0415
14	4.0747	4.6604	5.6287	6.5706	7.7895
15	4.6009	5.2294	6.2621	7.2609	8.5468
16	5.1422	5.8122	6.9077	7.9616	9.3122
17	5.6972	6.4078	7.5642	8.6718	10.0852
18	6.2648	7.0149	8.2308	9.3905	10.8649
19	6.8440	7.6327	8.9066	10.1170	11.6509
20	7.4339	8.2604	9.5908	10.8508	12.4426
21	8.0337	8.8972	10.2829	11.5913	13.2396
22	8.6427	9.5425	10.9823	12.3380	14.0415
23	9.2604	10.1957	11.6885	13.0905	14.8479
24	9.8862	10.8564	12.4011	13.8484	15.6587
25	10.5197	11.5240	13.1197	14.6114	16.4734
26	11.1603	12.1981	13.8439	15.3791	17.2919
27	11.8076	12.8786	14.5733	16.1513	18.1138
28	12.4613	13.5648	15.3079	16.9279	18.9392
29	13.1211	14.2565	16.0471	17.7083	19.7677
30	13.7867	14.9535	16.7908	18.4926	20.5992
40	20.7065	22.1643	24.4331	26.5093	29.0505
50	27.9907	29.7067	32.3574	34.7642	37.6886
60	35.5346	37.4848	40.4817	43.1879	46.4589
70	43.2752	45.4418	48.7576	51.7393	55.3290
80	51.1720	53.5400	57.1532	60.3915	64.2778
90	59.1963	61.7541	65.6466	69.1260	73.2912
100*	67.3276	70.0648	74.2219	77.9295	82.3581

*For d.f. greater than 100 the expression $\sqrt{2\chi^2} - \sqrt{(2k-1)} = Z$ follows the standardized normal distribution, where k represents the degrees of freedom.

.750	.500	.250	.100	.050	.025	.010	.005
.1015	.4549	1.3233	2.7055	3.8415	5.0239	6.6349	7.8794
.5754	1.3863	2.7726	4.6052	5.9915	7.3778	9.2103	10.5966
1.2125	2.3660	4.1084	6.2514	7.8147	9.3484	11.3449	12.8381
1.9226	3.3567	5.3853	7.7794	9.4877	11.1433	13.2767	14.8602
2.6746	4.3515	6.6257	9.2364	11.0705	12.8325	15.0863	16.7496
3.4546	5.3481	7.8408	10.6446	12.5916	14.4494	16.8119	18.5476
4.2549	6.3458	9.0372	12.0170	14.0671	16.0128	18.4753	20.2777
5.0706	7.3441	10.2188	13.3616	15.5073	17.5346	20.0902	21.9550
5.8988	8.3428	11.3887	14.6837	16.9190	19.0228	21.6660	23.5893
6.7372	9.3418	12.5489	15.9871	18.3070	20.4831	23.2093	25.1882
7.5841	10.3410	13.7007	17.2750	19.6751	21.9200	24.7250	26.7569
8.4384	11.3403	14.8454	18.5494	21.0261	23.3367	26.2170	28.2995
9.2991	12.3398	15.9839	19.8119	22.3621	24.7356	27.6883	29.8194
10.1653	13.3393	17.1170	21.0642	23.6848	26.1190	29.1413	31.3193
11.0365	14.3389	18.2451	22.3072	24.9958	27.4884	30.5779	32.8013
11.9122	15.3385	19.3688	23.5418	26.2962	28.8454	31.9999	34.2672
12.7919	16.3381	20.4887	24.7690	27.5871	30.1910	33.4087	35.7185
13.6753	17.3379	21.6049	25.9894	28.8693	31.5264	34.8053	37.1564
14.5620	18.3376	22.7178	27.2036	30.1435	32.8523	36.1908	38.5822
15.4518	19.3374	23.8277	28.4120	31.4104	34.1696	37.5662	39.9968
16.3444	20.3372	24.9348	29.6151	32.6705	35.4789	38.9321	41.4010
17.2396	21.3370	26.0393	30.8133	33.9244	36.7807	40.2894	42.7956
18.1373	22.3369	27.1413	32.0069	35.1725	38.0757	41.6384	44.1813
19.0372	23.3367	28.2412	33.1963	36.4151	39.3641	42.9798	45.5585
19.9393	24.3366	29.3389	34.3816	37.6525	40.6465	44.3141	46.9278
20.8434	25.3364	30.4345	35.5631	38.8852	41.9232	45.6417	48.2899
21.7494	26.3363	31.5284	36.7412	40.1133	43.1944	46.9630	49.6449
22.6572	27.3363	32.6205	37.9159	41.3372	44.4607	48.2782	50.9933
23.5666	28.3362	33.7109	39.0875	42.5569	45.7222	49.5879	52.3356
24.4776	29.3360	34.7998	40.2560	43.7729	46.9792	50.8922	53.6720
33.6603	39.3354	45.6160	51.8050	55.7585	59.3417	63.6907	66.7659
42.9421	49.3349	56.3336	63.1671	67.5048	71.4202	76.1539	79.4900
52.2938	59.3347	66.9814	74.3970	79.0819	83.2976	88.3794	91.9517
61.6983	69.3344	77.5766	85.5271	90.5312	95.0231	100.425	104.215
71.1445	79.3343	88.1303	96.5782	101.879	106.629	112.329	116.321
80.6247	89.3342	98.6499	107.565	113.145	118.136	124.116	128.299
90.1332	99.3341	109.141	118.498	124.342	129.561	135.807	140.169

Source: Abridged from E. S. Pearson and H. O. Hartley, eds., *Biometrika Tables for Statisticians*, vol. 1, 3rd ed., Table 8, Cambridge University Press, New York, 1966. Reproduced by permission of the editors and trustees of *Biometrika*.

TABLE E-5a DURBIN-WATSON d STATISTIC: SIGNIFICANCE POINTS OF d_L AND d_U AT 0.05 LEVEL OF SIGNIFICANCE

	$k'=1$		$k'=2$		$k'=3$		$k'=4$		$k'=5$		$k'=6$		$k'=7$		$k'=8$		$k'=9$		$k'=10$	
n	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U
6	0.610	1.400	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
7	0.700	1.356	0.467	1.896	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
8	0.763	1.332	0.559	1.777	0.368	2.287	—	—	—	—	—	—	—	—	—	—	—	—	—	—
9	0.824	1.320	0.629	1.699	0.455	2.128	0.296	2.588	—	—	—	—	—	—	—	—	—	—	—	—
10	0.879	1.320	0.697	1.641	0.525	2.016	0.376	2.414	0.243	2.822	—	—	—	—	—	—	—	—	—	—
11	0.927	1.324	0.658	1.604	0.595	1.928	0.444	2.283	0.316	2.645	0.203	3.005	—	—	—	—	—	—	—	—
12	0.971	1.331	0.812	1.579	0.658	1.864	0.512	2.177	0.379	2.506	0.268	2.832	0.171	3.149	—	—	—	—	—	—
13	1.010	1.340	0.861	1.562	0.715	1.816	0.574	2.094	0.445	2.390	0.328	2.692	0.230	2.985	0.147	3.266	—	—	—	—
14	1.045	1.350	0.905	1.551	0.767	1.779	0.632	2.030	0.505	2.296	0.389	2.572	0.286	2.848	0.200	3.111	0.127	3.360	—	—
15	1.077	1.361	0.946	1.543	0.814	1.750	0.685	1.977	0.562	2.220	0.447	2.472	0.343	2.727	0.251	2.979	0.175	3.216	0.111	3.438
16	1.106	1.371	0.982	1.539	0.857	1.728	0.734	1.935	0.615	2.157	0.502	2.388	0.398	2.624	0.304	2.860	0.222	3.090	0.155	3.304
17	1.133	1.381	1.015	1.536	0.897	1.710	0.779	1.900	0.664	2.104	0.554	2.318	0.451	2.537	0.356	2.757	0.272	2.975	0.198	3.184
18	1.158	1.391	1.046	1.535	0.933	1.696	0.820	1.872	0.710	2.060	0.603	2.257	0.502	2.461	0.407	2.667	0.321	2.873	0.244	3.073
19	1.180	1.401	1.074	1.536	0.967	1.685	0.859	1.848	0.752	2.023	0.649	2.206	0.549	2.396	0.456	2.589	0.369	2.783	0.290	2.974
20	1.201	1.411	1.100	1.537	0.998	1.676	0.894	1.828	0.792	1.991	0.692	2.162	0.595	2.339	0.502	2.521	0.416	2.704	0.336	2.885
21	1.221	1.420	1.125	1.538	1.026	1.669	0.927	1.812	0.829	1.964	0.732	2.124	0.637	2.290	0.547	2.460	0.461	2.633	0.380	2.806
22	1.239	1.429	1.147	1.541	1.053	1.664	0.958	1.797	0.863	1.940	0.769	2.090	0.677	2.246	0.588	2.407	0.504	2.571	0.424	2.734
23	1.257	1.437	1.168	1.543	1.078	1.660	0.986	1.785	0.895	1.920	0.804	2.061	0.715	2.208	0.628	2.360	0.545	2.514	0.465	2.670
24	1.273	1.446	1.188	1.546	1.101	1.656	1.013	1.775	0.925	1.902	0.837	2.035	0.751	2.174	0.666	2.318	0.584	2.464	0.506	2.613
25	1.288	1.454	1.206	1.550	1.123	1.654	1.038	1.767	0.953	1.886	0.868	2.012	0.784	2.144	0.702	2.280	0.621	2.419	0.544	2.560
26	1.302	1.461	1.224	1.553	1.143	1.652	1.062	1.759	0.979	1.873	0.897	1.992	0.816	2.117	0.735	2.246	0.657	2.379	0.581	2.513
27	1.316	1.469	1.240	1.556	1.162	1.651	1.084	1.753	1.004	1.861	0.925	1.974	0.845	2.093	0.767	2.216	0.691	2.342	0.616	2.470
28	1.328	1.476	1.255	1.560	1.181	1.650	1.104	1.747	1.028	1.850	0.951	1.958	0.874	2.071	0.798	2.188	0.723	2.309	0.650	2.431
29	1.341	1.483	1.270	1.563	1.198	1.650	1.124	1.743	1.050	1.841	0.975	1.944	0.900	2.052	0.826	2.164	0.753	2.278	0.682	2.396
30	1.352	1.489	1.284	1.567	1.214	1.650	1.143	1.739	1.071	1.833	0.998	1.931	0.926	2.034	0.854	2.141	0.782	2.251	0.712	2.363
31	1.363	1.496	1.297	1.570	1.229	1.650	1.160	1.735	1.090	1.825	1.020	1.920	0.950	2.018	0.879	2.120	0.810	2.226	0.741	2.333
32	1.373	1.502	1.309	1.574	1.244	1.650	1.177	1.732	1.109	1.819	1.041	1.909	0.972	2.004	0.904	2.102	0.836	2.203	0.769	2.306
33	1.383	1.508	1.321	1.577	1.258	1.651	1.193	1.730	1.127	1.813	1.061	1.900	0.994	1.991	0.927	2.085	0.861	2.181	0.795	2.281
34	1.393	1.514	1.333	1.580	1.271	1.652	1.208	1.728	1.144	1.808	1.080	1.891	1.015	1.979	0.950	2.069	0.885	2.162	0.821	2.257
35	1.402	1.519	1.343	1.584	1.283	1.653	1.222	1.726	1.160	1.803	1.097	1.884	1.034	1.967	0.971	2.054	0.908	2.144	0.845	2.236
36	1.411	1.525	1.354	1.587	1.295	1.654	1.236	1.724	1.175	1.799	1.114	1.877	1.053	1.957	0.991	2.041	0.930	2.127	0.868	2.216
37	1.419	1.530	1.364	1.590	1.307	1.655	1.249	1.723	1.190	1.795	1.131	1.870	1.071	1.948	1.011	2.029	0.951	2.112	0.891	2.198
38	1.427	1.535	1.373	1.594	1.318	1.656	1.261	1.722	1.204	1.792	1.146	1.864	1.088	1.939	1.029	2.017	0.970	2.098	0.912	2.180
39	1.435	1.540	1.382	1.597	1.328	1.658	1.273	1.722	1.218	1.789	1.161	1.859	1.104	1.932	1.047	2.007	0.990	2.085	0.932	2.164
40	1.442	1.544	1.391	1.600	1.338	1.659	1.285	1.721	1.230	1.786	1.175	1.854	1.120	1.924	1.064	1.997	1.008	2.072	0.952	2.149
45	1.475	1.566	1.430	1.615	1.383	1.666	1.336	1.720	1.287	1.776	1.238	1.835	1.189	1.895	1.139	1.958	1.089	2.022	1.038	2.088
50	1.503	1.585	1.462	1.628	1.421	1.674	1.378	1.721	1.335	1.771	1.291	1.822	1.246	1.875	1.201	1.930	1.156	1.986	1.110	2.044
55	1.528	1.601	1.490	1.641	1.452	1.681	1.414	1.724	1.374	1.768	1.334	1.814	1.294	1.861	1.253	1.909	1.212	1.959	1.170	2.010
60	1.549	1.616	1.514	1.652	1.480	1.689	1.444	1.727	1.408	1.767	1.372	1.808	1.335	1.850	1.298	1.894	1.260	1.939	1.222	1.984
65	1.567	1.629	1.536	1.662	1.503	1.696	1.471	1.731	1.438	1.767	1.404	1.805	1.370	1.843	1.336	1.882	1.301	1.923	1.266	1.964
70	1.583	1.641	1.554	1.672	1.525	1.703	1.494	1.735	1.464	1.768	1.433	1.802	1.401	1.837	1.369	1.873	1.337	1.910	1.305	1.948
75	1.598	1.652	1.571	1.680	1.543	1.709	1.515	1.739	1.487	1.770	1.458	1.801	1.428	1.834	1.399	1.867	1.369	1.901	1.339	1.935
80	1.611	1.662	1.586	1.688	1.560	1.715	1.534	1.743	1.507	1.772	1.480	1.801	1.453	1.831	1.425	1.861	1.397	1.893	1.369	1.925
85	1.624	1.671	1.600	1.696	1.575	1.721	1.550	1.747	1.525	1.774	1.500	1.801	1.474	1.829	1.448	1.857	1.422	1.886	1.396	1.916
90	1.635	1.679	1.612	1.703	1.589	1.726	1.566	1.751	1.542	1.776	1.518	1.801	1.494	1.827	1.469	1.854	1.445	1.881	1.420	1.909
95	1.645	1.687	1.623	1.709	1.602	1.732	1.579	1.755	1.557	1.778	1.535	1.802	1.512	1.827	1.489	1.852	1.465	1.877	1.442	1.903
100	1.654	1.694	1.634	1.715	1.613	1.736	1.592	1.758	1.571	1.780	1.550	1.803	1.528	1.826	1.506	1.850	1.484	1.874	1.462	1.898
150	1.720	1.746	1.706	1.760	1.693	1.774	1.679	1.788	1.665	1.802	1.651	1.817	1.637	1.832	1.622	1.847	1.608	1.862	1.594	1.877
200	1.758	1.778	1.748	1.789	1.738	1.799	1.728	1.810	1.718	1.820	1.707	1.831	1.697	1.841	1.686	1.852	1.675	1.863	1.665	1.874

	$k'=11$		$k'=12$		$k'=13$		$k'=14$		$k'=15$		$k'=16$		$k'=17$		$k'=18$		$k'=19$		$k'=20$	
n	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U
16	0.098	3.503	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
17	0.138	3.378	0.087	3.557	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
18	0.177	3.265	0.123	3.441	0.078	3.603	—	—	—	—	—	—	—	—	—	—	—	—	—	—
19	0.220	3.159	0.160	3.335	0.111	3.496	0.070	3.642	—	—	—	—	—	—	—	—	—	—	—	—
20	0.263	3.063	0.200	3.234	0.145	3.395	0.100	3.542	0.063	3.676	—	—	—	—	—	—	—	—	—	—
21	0.307	2.976	0.240	3.141	0.182	3.300	0.132	3.448	0.091	3.583	0.058	3.705	—	—	—	—	—	—	—	—
22	0.349	2.897	0.281	3.057	0.220	3.211	0.166	3.358	0.120	3.495	0.083	3.619	0.052	3.731	—	—	—	—	—	—
23	0.391	2.825	0.322	2.979	0.259	3.128	0.202	3.272	0.153	3.409	0.110	3.535	0.076	3.650	0.048	3.753	—	—	—	—
24	0.431	2.761	0.362	2.908	0.297	3.053	0.239	3.193	0.186	3.327	0.141	3.454	0.101	3.572	0.070	3.678	0.044	3.773	—	—
25	0.470	2.702	0.400	2.844	0.335	2.983	0.275	3.119	0.221	3.251	0.172	3.376	0.130	3.494	0.094	3.604	0.065	3.702	0.041	3.790
26	0.508	2.649	0.438	2.784	0.373	2.919	0.312	3.051	0.256	3.179	0.205	3.303	0.160	3.420	0.120	3.531	0.087	3.632	0.060	3.724
27	0.544	2.600	0.475	2.730	0.409	2.859	0.348	2.987	0.291	3.112	0.238	3.233	0.191	3.349	0.149	3.460	0.112	3.563	0.081	3.658
28	0.578	2.555	0.510	2.680	0.445	2.805	0.383	2.928	0.325	3.050	0.271	3.168	0.222	3.283	0.178	3.392	0.138	3.495	0.104	3.592
29	0.612	2.515	0.544	2.634	0.479	2.755	0.418	2.874	0.359	2.992	0.305	3.107	0.254	3.219	0.208	3.327	0.166	3.431	0.129	3.528
30	0.643	2.477	0.577	2.592	0.512	2.708	0.451	2.823	0.392	2.937	0.337	3.050	0.286	3.160	0.238	3.266	0.195	3.368	0.156	3.465
31	0.674	2.443	0.608	2.553	0.545	2.665	0.484	2.776	0.425	2.887	0.370	2.996	0.317	3.103	0.269	3.208	0.224	3.309	0.183	3.406
32	0.703	2.411	0.638	2.517	0.576	2.625	0.515	2.733	0.457	2.840	0.401	2.946	0.349	3.050	0.299	3.153	0.253	3.252	0.211	3.348
33	0.731	2.382	0.668	2.484	0.606	2.588	0.546	2.692	0.488	2.796	0.432	2.899	0.379	3.000	0.329	3.100	0.283	3.198	0.239	3.293
34	0.758	2.355	0.695	2.454	0.634	2.554	0.575	2.654	0.518	2.754	0.462	2.854	0.409	2.954	0.359	3.051	0.312	3.147	0.267	3.240
35	0.783	2.330	0.722	2.425	0.662	2.521	0.604	2.619	0.547	2.716	0.492	2.813	0.439	2.910	0.388	3.005	0.340	3.099	0.295	3.190
36	0.808	2.306	0.748	2.398	0.689	2.492	0.631	2.586	0.575	2.680	0.520	2.774	0.467	2.868	0.417	2.961	0.369	3.053	0.323	3.142
37	0.831	2.285	0.772	2.374	0.714	2.464	0.657	2.555	0.602	2.646	0.548	2.738	0.495	2.829	0.445	2.920	0.397	3.009	0.351	3.097
38	0.854	2.265	0.796	2.351	0.739	2.438	0.683	2.526	0.628	2.614	0.575	2.703	0.522	2.792	0.472	2.880	0.424	2.968	0.378	3.054
39	0.875	2.246	0.819	2.329	0.763	2.413	0.707	2.499	0.653	2.585	0.600	2.671	0.549	2.757	0.499	2.843	0.451	2.929	0.404	3.013
40	0.896	2.228	0.840	2.309	0.785	2.391	0.731	2.473	0.678	2.557	0.626	2.641	0.575	2.724	0.525	2.808	0.477	2.892	0.430	2.974
45	0.988	2.156	0.938	2.225	0.887	2.296	0.838	2.367	0.788	2.439	0.740	2.512	0.692	2.586	0.644	2.659	0.598	2.733	0.553	2.807
50	1.064	2.103	1.019	2.163	0.973	2.225	0.927	2.287	0.882	2.350	0.836	2.414	0.792	2.479	0.747	2.544	0.703	2.610	0.660	2.675
55	1.129	2.062	1.087	2.116	1.045	2.170	1.003	2.225	0.961	2.281	0.919	2.338	0.877	2.396	0.836	2.454	0.795	2.512	0.754	2.571
60	1.184	2.031	1.145	2.079	1.108	2.127	1.068	2.177	1.029	2.227	0.990	2.278	0.951	2.330	0.913	2.382	0.874	2.434	0.836	2.487
65	1.231	2.006	1.195	2.049	1.160	2.093	1.124	2.138	1.088	2.183	1.052	2.229	1.016	2.276	0.980	2.323	0.944	2.371	0.908	2.419
70	1.272	1.986	1.239	2.026	1.206	2.066	1.172	2.106	1.139	2.148	1.105	2.189	1.072	2.232	1.038	2.275	1.005	2.318	0.971	2.362
75	1.308	1.970	1.277	2.008	1.247	2.043	1.215	2.080	1.184	2.118	1.153	2.156	1.121	2.195	1.090	2.235	1.058	2.275	1.027	2.315
80	1.340	1.957	1.311	1.991	1.283	2.024	1.253	2.059	1.224	2.093	1.195	2.129	1.165	2.165	1.136	2.201	1.106	2.238	1.076	2.275
85	1.369	1.946	1.342	1.977	1.315	2.009	1.287	2.040	1.260	2.073	1.232	2.105	1.205	2.139	1.177	2.172	1.149	2.206	1.121	2.241
90	1.395	1.937	1.369	1.966	1.344	1.995	1.318	2.025	1.292	2.055	1.266	2.085	1.240	2.116	1.213	2.148	1.187	2.179	1.160	2.211
95	1.418	1.929	1.394	1.956	1.370	1.984	1.345	2.012	1.321	2.040	1.296	2.068	1.271	2.097	1.247	2.126	1.222	2.156	1.197	2.186
100	1.439	1.923	1.416	1.948	1.393	1.974	1.371	2.000	1.347	2.026	1.324	2.053	1.301	2.080	1.277	2.108	1.253	2.135	1.229	2.164
150	1.579	1.892	1.564	1.908	1.550	1.924	1.535	1.940	1.519	1.956	1.504	1.972	1.489	1.989	1.474	2.006	1.458	2.023	1.443	2.040
200	1.654	1.885	1.643	1.896	1.632	1.908	1.621	1.919	1.610	1.931	1.599	1.943	1.588	1.955	1.576	1.967	1.565	1.979	1.554	1.991

Note: n = number of observations, k' = number of explanatory variables excluding the constant term.

Source: This table is an extension of the original Durbin-Watson table and is reproduced from N. E. Savin and K. J. White, "The Durbin-Watson Test for Serial Correlation with Extreme Small Samples or Many Regressors," *Econometrica*, vol. 45, November 1977, pp. 1989–96 and as corrected by R. W. Farebrother, *Econometrica*, vol. 48, September 1980, p. 1554. Reprinted by permission of the Econometric Society.

Example E.1.

If $n = 40$ and $k' = 4$, $d_L = 1.285$ and $d_U = 1.721$. If a computed d value is less than 1.285, there is evidence of positive first-order serial correlation; if it is greater than 1.721, there is no evidence of positive first-order serial correlation; but if d lies between the lower and the upper limit, there is inconclusive evidence regarding the presence or absence of positive first-order serial correlation.

TABLE E-5b DURBIN-WATSON d STATISTIC: SIGNIFICANCE POINTS OF d_L AND d_U AT 0.01 LEVEL OF SIGNIFICANCE

	$k'=1$		$k'=2$		$k'=3$		$k'=4$		$k'=5$		$k'=6$		$k'=7$		$k'=8$		$k'=9$		$k'=10$	
n	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U
6	0.390	1.142	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
7	0.435	1.036	0.294	1.676	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
8	0.497	1.003	0.345	1.489	0.229	2.102	—	—	—	—	—	—	—	—	—	—	—	—	—	—
9	0.554	0.998	0.408	1.389	0.279	1.875	0.183	2.433	—	—	—	—	—	—	—	—	—	—	—	—
10	0.604	1.001	0.466	1.333	0.340	1.733	0.230	2.193	0.150	2.690	—	—	—	—	—	—	—	—	—	—
11	0.653	1.010	0.519	1.297	0.396	1.640	0.286	2.030	0.193	2.453	0.124	2.892	—	—	—	—	—	—	—	—
12	0.697	1.023	0.569	1.274	0.449	1.575	0.339	1.913	0.244	2.280	0.164	2.665	0.105	3.053	—	—	—	—	—	—
13	0.738	1.038	0.616	1.261	0.499	1.526	0.391	1.826	0.294	2.150	0.211	2.490	0.140	2.838	0.090	3.182	—	—	—	—
14	0.776	1.054	0.660	1.254	0.547	1.490	0.441	1.757	0.343	2.049	0.257	2.354	0.183	2.667	0.122	2.981	0.078	3.287	—	—
15	0.811	1.070	0.700	1.252	0.591	1.464	0.488	1.704	0.391	1.967	0.303	2.244	0.226	2.530	0.161	2.817	0.107	3.101	0.068	3.374
16	0.844	1.086	0.737	1.252	0.633	1.446	0.532	1.663	0.437	1.900	0.349	2.153	0.269	2.416	0.200	2.681	0.142	2.944	0.094	3.201
17	0.874	1.102	0.772	1.255	0.672	1.432	0.574	1.630	0.480	1.847	0.393	2.078	0.313	2.319	0.241	2.566	0.179	2.811	0.127	3.053
18	0.902	1.118	0.805	1.259	0.708	1.422	0.613	1.604	0.522	1.803	0.435	2.015	0.355	2.238	0.282	2.467	0.216	2.697	0.160	2.925
19	0.928	1.132	0.835	1.265	0.742	1.415	0.650	1.584	0.561	1.767	0.476	1.963	0.396	2.169	0.322	2.381	0.255	2.597	0.196	2.813
20	0.952	1.147	0.863	1.271	0.773	1.411	0.685	1.567	0.598	1.737	0.515	1.918	0.436	2.110	0.362	2.308	0.294	2.510	0.232	2.714
21	0.975	1.161	0.890	1.277	0.803	1.408	0.718	1.554	0.633	1.712	0.552	1.881	0.474	2.059	0.400	2.244	0.331	2.434	0.268	2.625
22	0.997	1.174	0.914	1.284	0.831	1.407	0.748	1.543	0.667	1.691	0.587	1.849	0.510	2.015	0.437	2.188	0.368	2.367	0.304	2.548
23	1.018	1.187	0.938	1.291	0.858	1.407	0.777	1.534	0.698	1.673	0.620	1.821	0.545	1.977	0.473	2.140	0.404	2.308	0.340	2.479
24	1.037	1.199	0.960	1.298	0.882	1.407	0.805	1.528	0.728	1.658	0.652	1.797	0.578	1.944	0.507	2.097	0.439	2.255	0.375	2.417
25	1.055	1.211	0.981	1.305	0.906	1.409	0.831	1.523	0.756	1.645	0.682	1.776	0.610	1.915	0.540	2.059	0.473	2.209	0.409	2.362
26	1.072	1.222	1.001	1.312	0.928	1.411	0.855	1.518	0.783	1.635	0.711	1.759	0.640	1.889	0.572	2.026	0.505	2.168	0.441	2.313
27	1.089	1.233	1.019	1.319	0.949	1.413	0.878	1.515	0.808	1.626	0.738	1.743	0.669	1.867	0.602	1.997	0.536	2.131	0.473	2.269
28	1.104	1.244	1.037	1.325	0.969	1.415	0.900	1.513	0.832	1.618	0.764	1.729	0.696	1.847	0.630	1.970	0.566	2.098	0.504	2.229
29	1.119	1.254	1.054	1.332	0.988	1.418	0.921	1.512	0.855	1.611	0.788	1.718	0.723	1.830	0.658	1.947	0.595	2.068	0.533	2.193
30	1.133	1.263	1.070	1.339	1.006	1.421	0.941	1.511	0.877	1.606	0.812	1.707	0.748	1.814	0.684	1.925	0.622	2.041	0.562	2.160
31	1.147	1.273	1.085	1.345	1.023	1.425	0.960	1.510	0.897	1.601	0.834	1.698	0.772	1.800	0.710	1.906	0.649	2.017	0.589	2.131
32	1.160	1.282	1.100	1.352	1.040	1.428	0.979	1.510	0.917	1.597	0.856	1.690	0.794	1.788	0.734	1.889	0.674	1.995	0.615	2.104
33	1.172	1.291	1.114	1.358	1.055	1.432	0.996	1.510	0.936	1.594	0.876	1.683	0.816	1.776	0.757	1.874	0.698	1.975	0.641	2.080
34	1.184	1.299	1.128	1.364	1.070	1.435	1.012	1.511	0.954	1.591	0.896	1.677	0.837	1.766	0.779	1.860	0.722	1.957	0.665	2.057
35	1.195	1.307	1.140	1.370	1.085	1.439	1.028	1.512	0.971	1.589	0.914	1.671	0.857	1.757	0.800	1.847	0.744	1.940	0.689	2.037
36	1.206	1.315	1.153	1.376	1.098	1.442	1.043	1.513	0.988	1.588	0.932	1.666	0.877	1.749	0.821	1.836	0.766	1.925	0.711	2.018
37	1.217	1.323	1.165	1.382	1.112	1.446	1.058	1.514	1.004	1.586	0.950	1.662	0.895	1.742	0.841	1.825	0.787	1.911	0.733	2.001
38	1.227	1.330	1.176	1.388	1.124	1.449	1.072	1.515	1.019	1.585	0.966	1.658	0.913	1.735	0.860	1.816	0.807	1.899	0.754	1.985
39	1.237	1.337	1.187	1.393	1.137	1.453	1.085	1.517	1.034	1.584	0.982	1.655	0.930	1.729	0.878	1.807	0.826	1.887	0.774	1.970
40	1.246	1.344	1.198	1.398	1.148	1.457	1.098	1.518	1.048	1.584	0.997	1.652	0.946	1.724	0.895	1.799	0.844	1.876	0.749	1.956
45	1.288	1.376	1.245	1.423	1.201	1.474	1.156	1.528	1.111	1.584	1.065	1.643	1.019	1.704	0.974	1.768	0.927	1.834	0.881	1.902
50	1.324	1.403	1.285	1.446	1.245	1.491	1.205	1.538	1.164	1.587	1.123	1.639	1.081	1.692	1.039	1.748	0.997	1.805	0.955	1.864
55	1.356	1.427	1.320	1.466	1.284	1.506	1.247	1.548	1.209	1.592	1.172	1.638	1.134	1.685	1.095	1.734	1.057	1.785	1.018	1.837
60	1.383	1.449	1.350	1.484	1.317	1.520	1.283	1.558	1.249	1.598	1.214	1.639	1.179	1.682	1.144	1.726	1.108	1.771	1.072	1.817
65	1.407	1.468	1.377	1.500	1.346	1.534	1.315	1.568	1.283	1.604	1.251	1.642	1.218	1.680	1.186	1.720	1.153	1.761	1.120	1.802
70	1.429	1.485	1.400	1.515	1.372	1.546	1.343	1.578	1.313	1.611	1.283	1.645	1.253	1.680	1.223	1.716	1.192	1.754	1.162	1.792
75	1.448	1.501	1.422	1.529	1.395	1.557	1.368	1.587	1.340	1.617	1.313	1.649	1.284	1.682	1.256	1.714	1.227	1.748	1.199	1.783
80	1.466	1.515	1.441	1.541	1.416	1.568	1.390	1.595	1.364	1.624	1.338	1.653	1.312	1.683	1.285	1.714	1.259	1.745	1.232	1.777
85	1.482	1.528	1.458	1.553	1.435	1.578	1.411	1.603	1.386	1.630	1.362	1.657	1.337	1.685	1.312	1.714	1.287	1.743	1.262	1.773
90	1.496	1.540	1.474	1.563	1.452	1.587	1.429	1.611	1.406	1.636	1.383	1.661	1.360	1.687	1.336	1.714	1.312	1.741	1.288	1.769
95	1.510	1.552	1.489	1.573	1.468	1.596	1.446	1.618	1.425	1.642	1.403	1.666	1.381	1.690	1.358	1.715	1.336	1.741	1.313	1.767
100	1.522	1.562	1.503	1.583	1.482	1.604	1.462	1.625	1.441	1.647	1.421	1.670	1.400	1.693	1.378	1.717	1.357	1.741	1.335	1.765
150	1.611	1.637	1.598	1.651	1.584	1.665	1.571	1.679	1.557	1.693	1.543	1.708	1.530	1.722	1.515	1.737	1.501	1.752	1.486	1.767
200	1.664	1.684	1.653	1.693	1.643	1.704	1.633	1.715	1.623	1.725	1.613	1.735	1.603	1.746	1.592	1.757	1.582	1.768	1.571	1.779

	$k'=11$		$k'=12$		$k'=13$		$k'=14$		$k'=15$		$k'=16$		$k'=17$		$k'=18$		$k'=19$		$k'=20$	
n	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U
16	0.060	3.446	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
17	0.094	3.286	0.053	3.506	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
18	0.113	3.146	0.075	3.358	0.047	3.357	—	—	—	—	—	—	—	—	—	—	—	—	—	—
19	0.145	3.023	0.102	3.227	0.067	3.420	0.043	3.601	—	—	—	—	—	—	—	—	—	—	—	—
20	0.178	2.914	0.131	3.109	0.092	3.297	0.061	3.474	0.038	3.639	—	—	—	—	—	—	—	—	—	—
21	0.212	2.817	0.162	3.004	0.119	3.185	0.084	3.358	0.055	3.521	0.035	3.671	—	—	—	—	—	—	—	—
22	0.245	2.729	0.194	2.909	0.148	3.084	0.109	3.252	0.077	3.412	0.050	3.562	0.032	3.700	—	—	—	—	—	—
23	0.281	2.651	0.227	2.822	0.178	2.991	0.136	3.155	0.100	3.311	0.070	3.459	0.046	3.597	0.029	3.725	—	—	—	—
24	0.315	2.580	0.260	2.744	0.209	2.906	0.165	3.065	0.125	3.218	0.092	3.363	0.065	3.501	0.043	3.629	0.027	3.747	—	—
25	0.348	2.517	0.292	2.674	0.240	2.829	0.194	2.982	0.152	3.131	0.116	3.274	0.085	3.410	0.060	3.538	0.039	3.657	0.025	3.766
26	0.381	2.460	0.324	2.610	0.272	2.758	0.224	2.906	0.180	3.050	0.141	3.191	0.107	3.325	0.079	3.452	0.055	3.572	0.036	3.682
27	0.413	2.409	0.356	2.552	0.303	2.694	0.253	2.836	0.208	2.976	0.167	3.113	0.131	3.245	0.100	3.371	0.073	3.490	0.051	3.602
28	0.444	2.363	0.387	2.499	0.333	2.635	0.283	2.772	0.237	2.907	0.194	3.040	0.156	3.169	0.122	3.294	0.093	3.412	0.068	3.524
29	0.474	2.321	0.417	2.451	0.363	2.582	0.313	2.713	0.266	2.843	0.222	2.972	0.182	3.098	0.148	3.220	0.114	3.338	0.087	3.450
30	0.503	2.283	0.447	2.407	0.393	2.533	0.342	2.659	0.294	2.785	0.249	2.909	0.208	3.032	0.171	3.152	0.137	3.267	0.107	3.379
31	0.531	2.248	0.475	2.367	0.422	2.487	0.371	2.609	0.322	2.730	0.277	2.851	0.234	2.970	0.196	3.087	0.160	3.201	0.128	3.311
32	0.558	2.216	0.503	2.330	0.450	2.446	0.399	2.563	0.350	2.680	0.304	2.797	0.261	2.912	0.221	3.026	0.184	3.137	0.151	3.246
33	0.585	2.187	0.530	2.296	0.477	2.408	0.426	2.520	0.377	2.633	0.331	2.746	0.287	2.858	0.246	2.969	0.209	3.078	0.174	3.184
34	0.610	2.160	0.556	2.266	0.503	2.373	0.452	2.481	0.404	2.590	0.357	2.699	0.313	2.808	0.272	2.915	0.233	3.022	0.197	3.126
35	0.634	2.136	0.581	2.237	0.529	2.340	0.478	2.444	0.430	2.550	0.383	2.655	0.339	2.761	0.297	2.865	0.257	2.969	0.221	3.071
36	0.658	2.113	0.605	2.210	0.554	2.310	0.504	2.410	0.455	2.512	0.409	2.614	0.364	2.717	0.322	2.818	0.282	2.919	0.244	3.019
37	0.680	2.092	0.628	2.186	0.578	2.282	0.528	2.379	0.480	2.477	0.434	2.576	0.389	2.675	0.347	2.774	0.306	2.872	0.268	2.969
38	0.702	2.073	0.651	2.164	0.601	2.256	0.552	2.350	0.504	2.445	0.458	2.540	0.414	2.637	0.371	2.733	0.330	2.828	0.291	2.923
39	0.723	2.055	0.673	2.143	0.623	2.232	0.575	2.323	0.528	2.414	0.482	2.507	0.438	2.600	0.395	2.694	0.354	2.787	0.315	2.879
40	0.744	2.039	0.694	2.123	0.645	2.210	0.597	2.297	0.551	2.386	0.505	2.476	0.461	2.566	0.418	2.657	0.377	2.748	0.338	2.838
45	0.835	1.972	0.790	2.044	0.744	2.118	0.700	2.193	0.655	2.269	0.612	2.346	0.570	2.424	0.528	2.503	0.488	2.582	0.448	2.661
50	0.913	1.925	0.871	1.987	0.829	2.051	0.787	2.116	0.746	2.182	0.705	2.250	0.665	2.318	0.625	2.387	0.586	2.456	0.548	2.526
55	0.979	1.891	0.940	1.945	0.902	2.002	0.863	2.059	0.825	2.117	0.786	2.176	0.748	2.237	0.711	2.298	0.674	2.359	0.637	2.421
60	1.037	1.865	1.001	1.914	0.965	1.964	0.929	2.015	0.893	2.067	0.857	2.120	0.822	2.173	0.786	2.227	0.751	2.283	0.716	2.338
65	1.087	1.845	1.053	1.889	1.020	1.934	0.986	1.980	0.953	2.027	0.919	2.075	0.886	2.123	0.852	2.172	0.819	2.221	0.786	2.272
70	1.131	1.831	1.099	1.870	1.068	1.911	1.037	1.953	1.005	1.995	0.974	2.038	0.943	2.082	0.911	2.127	0.880	2.172	0.849	2.217
75	1.170	1.819	1.141	1.856	1.111	1.893	1.082	1.931	1.052	1.970	1.023	2.009	0.993	2.049	0.964	2.090	0.934	2.131	0.905	2.172
80	1.205	1.810	1.177	1.844	1.150	1.878	1.122	1.913	1.094	1.949	1.066	1.984	1.039	2.022	1.011	2.059	0.983	2.097	0.955	2.135
85	1.236	1.803	1.210	1.834	1.184	1.866	1.158	1.898	1.132	1.931	1.106	1.965	1.080	1.999	1.053	2.033	1.027	2.068	1.000	2.104
90	1.264	1.798	1.240	1.827	1.215	1.856	1.191	1.886	1.166	1.917	1.141	1.948	1.116	1.979	1.091	2.012	1.066	2.044	1.041	2.077
95	1.290	1.793	1.267	1.821	1.244	1.848	1.221	1.876	1.197	1.905	1.174	1.934	1.150	1.963	1.126	1.993	1.102	2.023	1.079	2.054
100	1.314	1.790	1.292	1.816	1.270	1.841	1.248	1.868	1.225	1.895	1.203	1.922	1.181	1.949	1.158	1.977	1.136	2.006	1.113	2.034
150	1.473	1.783	1.458	1.799	1.444	1.814	1.429	1.830	1.414	1.847	1.400	1.863	1.385	1.880	1.370	1.897	1.355	1.913	1.340	1.931
200	1.561	1.791	1.550	1.801	1.539	1.813	1.528	1.824	1.518	1.836	1.507	1.847	1.495	1.860	1.484	1.871	1.474	1.883	1.462	1.896

Note: n = number of observations, k' = number of explanatory variables excluding the constant term.

Source: Savin and White, op. cit., by permission of Econometric Society.

[This question paper contains 20 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 5107 **H**

Unique Paper Code : 2272102401

Name of the Paper : Intermediate Microeconomics II

Name of the Course : **B.A. (H) Economics**

Semester : IV

Duration : 3 Hours Maximum Marks : 90

समय : 3 घण्टे पूर्णांक : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt any **five** questions.

3. **All** questions carry equal marks (**18** marks each).

4. Use of non-programmable scientific calculator is permitted.

5. Answers may be written either in English or Hindi; but the same medium should be used throughout the paper.

P.T.O.

छात्रों के लिए निर्देश

1. इस प्रश्न-पत्र के मिलते ही ऊपर दिए गए निर्धारित स्थान पर अपना अनुक्रमांक लिखिए ।
2. किन्हीं पाँच प्रश्नों के उत्तर दीजिये ।
3. सभी प्रश्नों के अंक समान हैं । (प्रत्येक 18 अंक)
4. गैर-प्रोग्रामयोग्य वैज्ञानिक कैलकुलेटर के उपयोग की अनुमति है ।
5. इस प्रश्न-पत्र का उत्तर अंग्रेजी या हिंदी किसी एक भाषा में दीजिए, लेकिन सभी उत्तरों का माध्यम एक ही होना चाहिए ।

1. (a) Suppose a monopolist's demand for his product is $P(q,A) = 200 - 2q + \sqrt{A}$ where q is the quantity of the product and A is total advertising expenditure. If the monopolist is producing at a constant marginal cost of ₹4, how much quantity will he sell at equilibrium and what will be the optimum level of advertising expenditure? Check for the second order conditions.

(b) Suppose a small town has only one large pen factory and it is the only source of job in the town. The pen factory is planning to hire both high and low skilled workers from the town. Its production function is $Q(L_h, L_l) = 12 \ln L_h + 6 \ln L_l$. The supply curves for the two types of workers are $w_h(L_h) = 2L_h$ and $w_l(L_l) = L_l$ and a pen sells at ₹12. Find the optimal number of each type of labour hired by the firm and respective wages? Calculate also the firm's profit maximizing level of output. Check for the second order conditions.

(c) A monopolist operates in two different markets with separate demand curves: $q_1(p_1, A_1) = p_1^{-2} A_1$, and $q_2(p_1, A_1) = p_2^{-2} A_2^{0.5}$, where q, p and A denote the quantity demanded, price and advertising expenditure in respective markets.

(i) Show that the monopolist faces the same price elasticity of demand in both the markets.

(ii) Calculate the advertising elasticity of demand in both the markets.

(iii) Calculate the advertising to sales ratio using the elasticities obtained in (i) and (ii). Show that the advertising to sales ratio is greater in the market where demand is more sensitive to advertising. (5+8+5)

2. (a) Consider an exchange economy with two agents- Gaurav and Jaya. The preferences of Gaurav and Jaya over good X and good Y are represented by the following utility functions:

$$U^G = X_G + \ln Y_G$$

$$U^J = X_J + 2\ln Y_J$$

Gaurav's initial endowments of X and Y are 6 units and 4 units respectively and Jaya is initially endowed with 4 unit of X and 2 units of Y.

(i) Find the equation of the contract curve and depict it in the Edgeworth box diagram.

(ii) Suppose that price of good X is normalized to 1 and price of good Y is P_Y. Compute the Walrasian equilibrium.

(iii) Suppose local authority is not happy with the above - mentioned allocation of resources between Gaurav and Jaya. How does the government ensure socially desirable (i.e. more equitable) Pareto efficient allocation? Explain intuitively in the light of Second Theorem of Welfare Economics.

(b) What is the difference between monopolistic competition and perfect competition in the long run? Explain with the help of a diagram.

(12+6)

3. (a) There are 2,000 residents in a housing society at Noida. The residents enjoy only two goods, festivals and food. Festivals are a public good, whereas food is a private good. The residents have identical preferences for the two goods, denoted by $u_i = \frac{\sqrt{x}}{10} + \frac{1}{2} y_i$, where x is total amount (measured in Rupees) spent on festivals and y_i is the money (measured in Rupees) spent on food by individual i. Assume that the marginal cost of provision of festival is Rs. 1.

(i) Calculate the marginal rate of substitution of food for festivals.

(ii) What is the Pareto optimal amount of money spent on festivals?

(b) The DDA would like to build a new public park at K block, R.K.Puram, if the authority can raise enough taxes to cover the cost, which is *100, 000* rupees. There are four families in the block. Each family's benefit from a public park is as follows:

$v_1 = 10{,}000$, $v_2 = 20{,}000$, $v_3 = 30{,}000$, and $v_4 = 40{,}000$. The authority wants to use the Clarke & Groves demand- revealing tax mechanism. Let T_i be the tax contribution of family i towards the park.

(i) If each family reports its true benefit, v_i, how much will each family have to pay, T_i? Calculate the net benefit of the 4th family.

(ii) Suppose 4th family misreports its benefit as 50,000, whereas the other families report their true benefits. Does the park get built? If so, how much does 4th family have to pay? Is 4th family's net benefit different from its net benefit in part (a)?

(iii) Suppose 4th family misreports its benefit as 30,000, whereas the other families report their true benefits. Does the park get built? If so, how much does 4th family have to pay. Is 4th family's net benefit different from its net benefit in part (a)?

(iv) Show that the Samuelson optimality condition is satisfied. (6+12)

4. Atul produces chemicals. His cost function is $C(x) = 10x^2 + 200x$, where x is the number of liters of chemicals produced. The market price for a liter of a chemical is As. 1000. Vijay is a baker. He is adversely affected by the noxious fumes emitted by Atul's production of chemicals. His cost function is $C(y) = y^2 - 160y + xy$, where y is the number of baked goods produced. The market price for a baked good is Rs. 40.

(a) How many liters of chemicals and how many baked goods will be produced in the market equilibrium?

(b) Suppose Atul and Vijay were to jointly maximize profits. How many liters of chemicals and how many baked goods will be produced?

(c) Compare the competitive market outcome from (a) and the joint profit maximization outcome from (b). Which outcome is Pareto optimal? Why?

(d) One of the solutions to the externality problem is Pigouvian taxes. In the above problem, who pays the tax? How does the Pigouvian tax solve the externality problem?

(e) Another solution to this problem is Coasian bargaining. Suppose Atul has the legal right to emit noxious fumes. How can they arrive at the Pareto optimal outcome? Explain intuitively.

(2+3+4+4+5)

5. (a) Consider a pure exchange economy with two individuals, A and B, and two goods, X and Y. The individuals have the following utility functions: $U_A(X_A, Y_A) = X_A Y_A$ and $U_B(X_B, Y_B) = X_B Y_B$. The initial endowments of both the goods are given by : $\omega_A = (3,4)$ and $\omega_B = (2,1)$.

(i) Show that the initial allocation is not Pareto Efficient.

(ii) Calculate the competitive equilibrium allocation and price.

(iii) Suppose a planner takes away 1 unit of each of the goods X and Y from A and gives them to B. Find the competitive equilibrium allocation and price.

(iv) Is the competitive allocation in (3) Pareto Superior to the one in (2)? Explain.

(b) Explain the Lindahl solution to the public good problem. What are the shortcomings of Lindahl solution? (12+6)

6. Consider a monopoly firm whose demand function and cost function are $P(q) = 30 - q$ and $TC(q) = q^2 - 2q + 12$ respectively.

(i) If the monopolist charges uniform price for her product what will be the equilibrium level of price, output and profit?

(ii) The monopolist wishes to reward her regular customers by offering a quantity discount. What are the prices and quantities produced for first and second block?

(iii) Will the quantity discount be a profitable strategy for the monopolist in the aforementioned problem? (6+6+6)

7. (a) Consider a monopoly seller who sells premium quality computer chips. The inverse demand function for the computer chips is given by $p(q) = 120 - q$, where q is the number of chips sold and p is the price per chip. The firm is able to distinguish between all the buyers and exercises

first degree price discrimination. Assume that the monopoly has a marginal cost function given by $MC(q) = 3q$.

(i) Find the optimum number of chips sold by the seller.

(ii) Find the consumer and producer surplus.

(iii) Show that there is no deadweight loss.

(b) Let there be a community of 2 individuals, Anubha (A) and Bikas (B), each with a utility function $u_i(x, y_i) = y_i - 12(a_i - x)^2$, where y is the numeraire private good and x is a public good, say rose shrubs, in the locality; $a_A = 13$ and, $a_B = 5$. Let the cost of provision of public good be Rs. 240 per unit.

(i) Which individual has a greater marginal benefit from the provision of public good?

(ii) Find the Pareto optimal number of rose shrubs in the community.

(iii) Calculate the total expenditure on the number of rose shrubs found in (ii). Also calculate the contribution of each individual if their per unit contribution equals their marginal benefits. (9+9)

1 (क) मान लीजिए कि किसी एकाधिकारवादी के उत्पाद की मांग $P(q,A) = 200 - 2q + \sqrt{A}$ है, जिसमें q उस उत्पाद की मात्रा को दर्शाता है और A विज्ञापन पर किया गया कुल व्यय है। यदि वह एकाधिकारवादी 4 रूपये की स्थिर सीमांत लागत पर उत्पादन कर रहा है, तो उसे संतुलन पर कितनी मात्रा में बिक्री करनी होगी और विज्ञापन पर किए जाने वाले व्यय का इष्टतम स्तर क्या होगा? सेकंड ऑर्डर कंडीशन की जाँच कीजिए।

(ख) मान लीजिए कि किसी छोटे शहर में पेन की केवल एक बड़ी फैक्ट्री है और यह शहर में नौकरी का एकमात्र स्रोत है। पेन फैक्ट्री उस शहर से उच्च कौशल प्राप्त और कम कुशल, दोनों तरह के श्रमिकों को काम पर रखने की योजना बना रही है। इसका उत्पादन फलन (function) $Q(L_h, L_l) = 12 \ln L_h + 6 \ln L_l$ है। इन दोनों प्रकार के श्रमिकों के लिए आपूर्ति वक्र $w_h(L_h) = 2L_h$ और $w_l(L_l) = L_l$ है। और एक पेन की बिक्री 12 रूपये पर होती है। फर्म द्वारा काम पर रखे गए प्रत्येक प्रकार के श्रमिकों की इष्टतम संख्या कितनी है और उनकी मजदूरी कितनी है? फर्म के उत्पादन के अधिकतम स्तर पर लाभ की भी गणना कीजिए। सेकंड ऑर्डर कंडीशन की जाँच कीजिए।

(ग) कोई एकाधिकारवादी दो अलग-अलग बाजारों में अलग-अलग मांग वक्रों, $q_1(p_1,A_1) = p_1^{-2}A_1$, और $q_2(p_1,A_1) = p_2^{-2} A_2^{0.5}$, के साथ काम करता है, जिसमें q, p और A संबंधित बाजारों में मांग की मात्रा, कीमत और विज्ञापन व्यय को दर्शाते हैं।

(i) दर्शाइए कि उस एकाधिकारवादी को दोनों ही बाजारों में माँग की समान कीमत लोच का सामना करना पड़ता है।

(ii) दोनों ही बाजारों में मांग की विज्ञापन लोच की गणना कीजिए।

(iii) उपरोक्त (i) और (ii) में प्राप्त लोच का उपयोग करते हुए विज्ञापन-बिक्री अनुपात की गणना कीजिए। यह दर्शाइए कि विज्ञापन-बिक्री का अनुपात उस बाजार में अधिक है, जिसमें माँग विज्ञापन के प्रति अधिक संवेदनशील है। (5+8+5)

2. (क) दो एजेंट-गौरव और जया के साथ एक विनिमय अर्थव्यवस्था पर विचार कीजिए। वस्तु X और वस्तु Y पर गौरव और जया की प्राथमिकताओं को निम्नलिखित उपयोगिता फलनों द्वारा दर्शाया गया है :

$$U^G = X_G + \ln Y_G$$

$$U^J = X_J + 2\ln Y_J$$

गौरव का X और Y का आरंभिक एंडाउमेंट क्रमशः 6 इकाइयाँ और 4 इकाइयाँ हैं और जया को शुरुआत में X की 4 इकाइयाँ और Y की 2 इकाइयाँ एंडाउ की जाती है।

(i) अनुबंध वक्र के समीकरण का पता लगाइए और इसे एजवर्थ बॉक्स डायग्राम में चित्रित कीजिए।

(ii) मान लीजिए कि वस्तु X की कीमत 1 पर सामान्यीकृत है और वस्तु Y की कीमत P_Y है। वालरासियन संतुलन की गणना कीजिए।

(iii) मान लीजिए कि स्थानीय प्राधिकारी गौरव और जया के बीच संसाधनों के उपर्युक्त आवंटन से खुश नहीं है। सरकार सामाजिक रूप से वांछनीय (यानी अधिक न्यायसंगत) पैरेटो कुशल आवंटन कैसे सुनिश्चित करती है? कल्याण अर्थशास्त्र के दूसरे सिद्धांत के प्रकाश में सहज रूप से समझाइए।

(ख) दीर्घकाल में एकाधिकार प्रतिस्पर्धा और उत्तम प्रतिस्पर्धा में क्या अंतर है? डायग्राम की सहायता से समझाइए। (12+6)

3. (क) नोएडा की एक हाउसिंग सोसायटी में 2,000 निवासी हैं। यहां के निवासी केवल दो वस्तुओं का आनंद लेते हैं, त्योहार और भोजन। त्यौहार एक सार्वजनिक वस्तु है, जबकि भोजन एक

निजी वस्तु है। निवासियों की दो वस्तुओं के लिए समान प्राथमिकताएँ हैं, जिन्हें $u_i = \frac{\sqrt{x}}{10} + \frac{1}{2} y_i$, से दर्शाया गया है। x त्योहारों पर खर्च की गई कुल राशि (रुपये में) है y_i व्यक्तियों i, द्वारा भोजन पर खर्च किया गया धन (रुपये में) है। मान लीजिए कि त्योहार के प्रावधान की सीमांत लागत 1 है।

(i) त्योहारों के लिए भोजन के प्रतिस्थापन की सीमांत दर (मार्जिनल रेट) की गणना कीजिए।

(ii) त्योहारों पर खर्च की जाने वाली पेरेटो इष्टतम राशि क्या है?

(ख) डीडीए, आर.के.पुरम के 'K' ब्लॉक में एक नया सार्वजनिक पार्क बनाना चाहेगा, यदि प्राधिकरण इस पर आने वाली लागत, जो कि 100,000 रुपये है, को पूरा करने के लिए कर की पर्याप्त वसूली करे। इस ब्लॉक में चार परिवार रहते हैं। सार्वजनिक पार्क से प्रत्येक परिवार को लाभ इस प्रकार है $v_1 = 10,000$, $v_2 = 20,000$, $v_3 = 30,000$, और $v_4 = 40,000$ प्राधिकरण क्लार्क एंड ग्रूव्स डिमांड रिविलिंग टैक्स मैकेनिज़्म का उपयोग करना चाहता है। T_i परिवार i द्वारा पार्क को दिया जाने वाला कर योगदान है।

(i) यदि प्रत्येक परिवार अपना वास्तविक लाभ, v_i, बताए तो प्रत्येक परिवार को कितना T_i का भुगतान करना होगा। चौथे परिवार के शुद्ध लाभ की गणना कीजिए।

(ii) मान लीजिए कि चौथा परिवार अपने लाभ को 50,000 के रूप में गलत बताता है, जबकि अन्य परिवार अपना वास्तविक लाभ बताते हैं। तो क्या पार्क बन सकता है? यदि हां, तो चौथे परिवार को कितना भुगतान करना होगा? क्या चौथे परिवार का शुद्ध लाभ भाग (ए) में इसके शुद्ध लाभ से भिन्न है?

(iii) मान लीजिए कि चौथा परिवार अपने लाभ को 30,000 के रूप में गलत बताता है, जबकि अन्य परिवार अपना वास्तविक लाभ बताते हैं। तो क्या पार्क बन सकता है? यदि हां, तो चौथे परिवार को कितना भुगतान करना होगा? क्या चौथे परिवार का शुद्ध लाभ भाग (ए) में इसके शुद्ध लाभ से भिन्न है?

(iv) दर्शाइए कि सैमुएलसन इष्टतमता की शर्तें पूरी होती है। (6+12)

4. अतुल रसायन का उत्पादन करता है। उसका लागत फलन $C(x) = 10x^2 + 200x$, है, जिसमें x उत्पादित किए गए रसायन की लीटर में संख्या है। एक लीटर रसायन का बाजार मूल्य 1000 रूपये है। विजय एक बेकर है। अतुल द्वारा रसायन के उत्पादन के दौरान फैक्ट्री से निकलने वाले जहरीले धुएं का उस पर प्रतिकूल असर पड़ता है। उसका लागत फलन $C(y) = y^2 - 160y + xy$, है। जिसमें y उत्पादित बेक किए गए माल की संख्या है। बेक किए गए माल का बाजार मूल्य 40 रूपये है।

(क) बाजार संतुलन में कितने लीटर रसायन और कितने बेक किए गए माल का उत्पादन होगा?

(ख) मान लीजिए अतुल और विजय को संयुक्त रूप से अपने लाभ को अधिकतम करना है। कितने लीटर रसायन और कितने बेक किए गए माल का उत्पादन किया जाएगा?

(ग) उपरोक्त (ए) से प्रतिस्पर्धी बाजार परिणामों और उपरोक्त (b) से संयुक्त लाभ अधिकतमकरण परिणामों की तुलना कीजिए। कौनसा परिणाम पेरेटो इष्टतम है? और क्यों?

(घ) बाहरी समस्या का एक समाधान पिगौवियन कर है। उपरोक्त समस्या में, टैक्स का भुगतान कौन करता है? पिगौवियन कर बाहरी समस्या का समाधान कैसे करता है?

(ङ) इस समस्या का एक अन्य समाधान कोएशियन सौदेबाजी है। मान लीजिए अतुल के पास हानिकारक धुंआ उत्सर्जित करने का कानूनी अधिकार है, तो वे किस प्रकार पेरेटो इष्टतम परिणाम पर पहुंच सकते हैं? सहजता से समझाइए।

(2+3+4+4+5)

5. (क) दो व्यक्तियों, A और B, और दो वस्तुओं, X और Y के साथ एक शुद्ध विनिमय अर्थव्यवस्था पर विचार कीजिए। व्यक्तियों के पास निम्नलिखित उपयोगिता फलन हैं : $U_A(X_A, Y_A) = X_A Y_A$ and $U_B(X_B, Y_B) = X_B Y_B$ दोनों वस्तुओं की आरंभिक बंदोबस्ती इस प्रकार दी गई है : $\omega_A = (3,4)$ और $\omega_B = (2,1)$

(i) दर्शाइए कि प्रारंभिक आवंटन पेरेटो कुशल नहीं है।

(ii) प्रतिस्पर्धी संतुलन आवंटन और मूल्य की गणना कीजिए।

(iii) मान लीजिए कोई योजनाकार A से प्रत्येक वस्तु X और Y की 1 इकाई लेता है और उन्हें B को दे देता है। प्रतिस्पर्धी मूल्य संतुलन आवंटन और मूल्य ज्ञात कीजिए।

(iv) क्या उपरोक्त (3) प्रतिस्पर्धी आवंटन, उपरोक्त (2) से पैरेटो सुपीरियर है? स्पष्ट कीजिए।

(ख) सार्वजनिक वस्तु संबंधी समस्या के लिए लिंडाहल समाधान को स्पष्ट कीजिए। लिंडाहल समाधान की कमियाँ बताइए?

(12 + 6)

6. किसी एकाधिकार फर्म पर विचार कीजिए, जिसका मांग फलन और लागत फलन क्रमश: $P(q) = 30 - q$ और $TC(q) = q^2 - 2q + 12$ है।

(i) यदि यह एकाधिकारवादी फर्म अपने उत्पाद के लिए एकसमान कीमत वसूले तो कीमत, उत्पादन और लाभ का संतुलन स्तर क्या होगा?

(ii) यह एकाधिकारवादी फर्म अपने नियमित ग्राहकों को मात्रा में छूट देकर फायदा देना चाहती है। तो पहले और दूसरे ब्लॉक के लिए कीमतें और उत्पादित किए गए माल की मात्रा कितनी है?

(iii) क्या उपर्युक्त समस्या में एकाधिकारवादी फर्म के लिए मात्रा में छूट देना लाभदायक रणनीति होगी? (6+6+6)

7. (क) किसी एकाधिकार वाले विक्रेता पर विचार कीजिए, जो प्रीमियम गुणवत्ता वाले कंप्यूटर चिप की बिक्री करता है। कंप्यूटर चिप के लिए व्युत्क्रम मांग फलन (फ़ंक्शन) $p(q) = 120 - q$, के माध्यम से दिया गया है, जिसमें q बिक्री की गई चिप की

संख्या को दर्शाता है और p प्रति चिप की कीमत को दर्शाता है। फर्म सभी खरीदारों के बीच अंतर करने में सक्षम है और फर्स्ट डिग्री प्राइस डिस्क्रिमिनेशन को लागू करती है। मान लीजिए कि इस मोनोपोली का सीमांत लागत फलन $MC(q) = 3q$ द्वारा दिया गया है।

(i) विक्रेता द्वारा बिक्री की गई अधिकतम चिप की संख्या ज्ञात कीजिए।

(ii) उपभोक्ता और उत्पादक अधिशेष ज्ञात कीजिए।

(iii) दर्शाइए कि कोई मृतभार घाटा (डेडवेट लॉस) नहीं हुआ है।

(ख) मान लीजिए कि दो व्यक्तियों, अनुभा (ए) और बिकास (बी) का एक समुदाय है, जिनमें से प्रत्येक का उपयोगिता फलन (utility function) है, $u_i(x, y_i) = y_i - 12(a_i - x)^2$ जिसमें y मूल्यमान (numeraire) निजी वस्तु है और x एक सार्वजनिक वस्तु है। मान लीजिए कि स्थानीय क्षेत्र में गुलाब की झाड़ियाँ $a_A = 13$ और, $a_B = 5$ है। माना सार्वजनिक वस्तु के प्रोविजन की लागत 240 रुपये प्रति इकाई है।

(i) सार्वजनिक वस्तु के प्रोविजन से किस व्यक्ति को अधिक सीमांत लाभ है?

(ii) समुदाय में गुलाब की झाड़ियों की पेरेटो इष्टतम संख्या ज्ञात कीजिए।

(iii) उपरोक्त (ii) में पाई गई गुलाब की झाड़ियों की संख्या पर किए गए कुल व्यय की गणना कीजिए। साथ ही, प्रत्येक व्यक्ति के योगदान की गणना कीजिए, यदि उनका प्रति इकाई योगदान उनके सीमांत लाभ के बराबर है। (9+9)

[This question paper contains 28 printed pages.]

Your Roll No...............

H

Sr. No. of Question Paper : 5213

Unique Paper Code : 2272102403

Name of the Paper : Introductory Econometrics

Name of the Course : **B.A. (Hons.) Economics (NEP-UGCF-2022)-DSCC-12**

Semester : IV

Duration : 3 Hour

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Question **1** is compulsory. Answer any **four** questions from the rest of the **six** questions. In total, **five** questions to be answered.
3. Use of simple calculator is permitted.
4. All intermediate calculations should be rounded off to four decimal places. The values provided in statistical tables should not be rounded off. All final calculations should be rounded off to two decimal places.
5. Answers may be written either in English or Hindi; but the same medium should be used throughout the paper.

छात्रों के लिए निर्देश

1. इस प्रश्न-पत्र के मिलते ही ऊपर दिए गए निर्धारित स्थान पर अपना अनुक्रमांक लिखिए।
2. प्रश्न 1 अनिवार्य है। शेष छह प्रश्नों में से किसी **चार** प्रश्नों के उत्तर दीजिए। कुल **पाँच** प्रश्नों के उत्तर देने हैं।
3. साधारण कैलकुलेटर उपयोग की अनुमति है।
4. सभी मध्यवर्ती गणनाएँ चार दशमलव स्थानों तक पूर्णांकित की जानी चाहिए। सांख्यिकीय तालिकाओं में दिए गए मानों को पूर्णांकित नहीं किया जाना चाहिए। सभी अंतिम गणनाएँ दो दशमलव स्थानों तक पूर्णांकित की जानी चाहिए।
5. इस प्रश्न-पत्र का उत्तर अंग्रेजी या हिंदी किसी एक भाषा में दीजिए, लेकिन सभी उत्तरों का माध्यम एक ही होना चाहिए।

1. State whether the following statements are true or false. Give reasons for your answer.

(a) For an OLS estimated regression equation, $\hat{Y}_i = b_1 + b_2X_i$, the sum of the product of the residuals and mean deviation of the explanatory variable is zero.

(b) Ceteris paribus, the higher the VIF (Variance Inflation Factor), the smaller is the variances of OLS estimators.

(c) For an OLS estimated regression equation, $\hat{Y}_i = b_1 + b_2X_i$, if we multiply both Y and X by 1000 and re-estimate two variable regression model, the intercept coefficient will increase by 1000 times.

(d) For a three variable regression model, TSS is always equal to the sum of ESS and RSS.

(e) It can easily happen that F statistic is significant while t statistics of explanatory variables are not significant.

(f) In a regression model with intercept, number of dummies for each qualitative variable must be one less than the number of categories of that variable. (3×6)

निम्नलिखित कथन सत्य हैं या असत्य। कारण सहित उत्तर दीजिए।

(क) एक ओएलएस (OLS) के लिए अनुमानित प्रतिगमन समीकरण $\hat{Y}_i = b_1 + b_2X_i$ है, अवशिष्ट के गुणनफल का योग और व्याख्यात्मक चर का माध्य विचलन शून्य है।

(ख) सेटेरिस परिबस (Ceteris paribus), VIF (विचरण मुद्रास्फीति कारक) जितना उच्च होगा, ओएलएस अनुमानक के प्रसरण उतने ही छोटे होंगे।

(ग) एक ओएलएस के लिए अनुमानित प्रतिगमन समीकरण $\hat{Y}_i = b_1 + b_2X_i$ है, यदि हम Y और X दोनों को 1000 से गुणा करते हैं और दो चर प्रतिगमन मॉडल का पुनः अनुमान लगाते हैं, तो अवरोधन गुणांक 1000 गुना बढ़ जाएगा।

(घ) तीन चर प्रतिगमन मॉडल के लिए, टीएसएस हमेशा ईएसएस और आरएसएस के योग के बराबर होता है।

(ङ) यह आसानी से हो सकता है कि F सांख्यिकी महत्वपूर्ण है जबकि व्याख्यात्मक चर की t सांख्यिकी महत्वपूर्ण नहीं हैं।

(च) अवरोधन के साथ एक प्रतिगमन मॉडल में, प्रत्येक गुणात्मक चर के लिए डमी की संख्या उस चर की श्रेणियों की संख्या से एक कम होनी चाहिए।

2. Using data for 110 schools the following model of monthly expenditure incurred by a school was estimated :

$$\widehat{\log(Y)}_i = 138.8 - 1.571\, D_i + 0.808\, X_i - 0.0944\, D_iX_i$$

$$t = \quad (2.864) \quad (-1.06) \quad (13.466) \quad (-2.04)$$

Y_i = annual expenditure incurred by a school, in thousands of rupees X_i = number of students in the school $Di = \begin{cases} 1 & \text{if government school} \\ 0 & \text{ifprivate school} \end{cases}$

(i) Write the regression equation for government school and private school separately. Also, interpret the slope coefficient of two categories.

(ii) Check the statistical significance of differential intercept coefficient and slope drifter at 10% level of significance. Based on the conclusion of these tests, draw the population regression lines for the government and private schools.

(iii) Test the hypothesis that the population error term is normally distributed at 1% level of significance when it is given that for the residuals, skewness is 0.28 and the value of kurtosis is 2. Also, in classical linear regression model, what is the rationale for the assumption that population error term follows normal distribution? (6,6,6)

110 स्कूलों के डेटा का उपयोग करते हुए, किसी स्कूल द्वारा किए गए मासिक व्यय के निम्नलिखित मॉडल का अनुमान लगाया गया था :

$$\widehat{\log(Y)}_i = 138.8 - 1.571\, D_i + 0.808\, X_i - 0.0944\, D_i X_i$$
$$t = \quad (2.864) \quad (-1.06) \quad (13.466) \quad (-2.04)$$

Y_i = स्कूल द्वारा किया गया वार्षिक व्यय, हजार रुपये में

X_i = स्कूल में विद्यार्थियों की संख्या

$$D_i = \begin{cases} 1 & \text{यदि सरकारी स्कूल है} \\ 0 & \text{यदि निजी स्कूल है} \end{cases}$$

(i) सरकारी स्कूल और निजी स्कूल के लिए अलग-अलग प्रतिगमन समीकरण लिखिए। इसके अलावा, दोनों श्रेणियों के ढलान गुणांक की व्याख्या कीजिए।

(ii) महत्व के 10% स्तर पर अंतर अवरोधन गुणांक और ढलान ड्रिफ्टर के सांख्यिकीय महत्व की जाँच कीजिए। इन परीक्षणों के निष्कर्ष के आधार पर, सरकारी और निजी स्कूलों के लिए जनसंख्या प्रतिगमन रेखाएँ खींचिए।

(iii) इस परिकल्पना का परीक्षण कीजिए कि जनसंख्या त्रुटि शब्द सामान्य रूप से महत्व के 1% स्तर पर वितरित किया जाता है जब अवशिष्टों, विषमता के लिए 0.28 है और पृथुशीर्षत्व का मान 2 दिया जाता है। इसके अलावा, शास्त्रीय रैखिक प्रतिगमन मॉडल में, इस धारणा के लिए तर्क क्या है कि जनसंख्या त्रुटि शब्द सामान्य वितरण का अनुसरण करता है?

3. (a) For estimating the Phillips curves for the United States from 1958 to 1969 the following regression was obtained :

$$\hat{Y}_t = -0.2594 + 20.5880 \frac{1}{X_t}$$
$$t = (-0.2572) \quad (4.3996)$$
$$R^2 = 0.6594, d = 0.6394$$

Y = Rate of change of money wages in percent

X = Unemployment rate in percent

(i) Interpret the intercept estimate. Is there any evidence of first order autocorrelation in the residuals at 5% level of significance?

(ii) Outline the method of estimation that will produce BLUE estimators in the presence of first order autocorrelation.

(b) Consider the following regression function on sales revenue for a particular firm for last 10 months, as estimated by OLS :

$$\hat{Y}_t = 51.508 + 0.054\,X_t$$
$$se = (4.03) \qquad (0.04)$$
$$\widehat{\sigma_u^2} = 1.02$$
$$\overline{X} = 25 \quad \sum X_t^2 = 6500$$

Y is the monthly sales revenue in billions of dollars

X is the monthly expenditure on advertising in millions of dollars.

Find the predicted mean sales revenue if the advertising expenditure for the firm in the next month is 30 million dollars. Also, find the 99% confidence interval for the true predicted mean of sales revenue corresponding to 30 million dollars advertising expenditure. (6+6,6)

(क) 1958 से 1969 तक संयुक्त राज्य अमेरिका के लिए फिलिप्स वक्र का अनुमान लगाने के लिए निम्नलिखित प्रतिगमन प्राप्त किया गया था :

$$\hat{Y}_t = -0.2594 + 20.5880\,\frac{1}{X_t}$$
$$t = (-0.2572) \quad (4.3996)$$
$$R^2 = 0.6594,\ d = 0.6394$$

Y = मुद्रा मजदूरी के परिवर्तन की दर प्रतिशत में

X = बेरोजगारी दर प्रतिशत में

(i) अवरोधन अनुमान की व्याख्या कीजिए। क्या 5% स्तर के महत्व पर अवशिष्टों में प्रथम क्रम के स्वत: सहसंबंध का कोई प्रमाण है?

(ii) पहले क्रम के स्वसहसंबंध की उपस्थिति में 'ब्लू' अनुमानक देने वाली अनुमान विधि की रूपरेखा तैयार कीजिए।

(ख) पिछले 10 महीनों के लिए किसी विशेष फर्म के लिए बिक्री राजस्व पर निम्नलिखित प्रतिगमन फलन पर विचार कीजिए, जैसा कि ओएलएस द्वारा अनुमानित है :

$$\hat{Y}_t = 51.508 + 0.054\, X_t$$

$$se = (4.03) \qquad (0.04)$$

$$\widehat{\sigma_u^2} = 1.02$$

$$\overline{X} = 25 \quad \sum X_t^2 = 6500$$

Y बिलियन डॉलर में मासिक बिक्री राजस्व है

X मिलियन डॉलर में विज्ञापन पर मासिक खर्च है

यदि अगले महीने में फर्म का विज्ञापन व्यय 30 मिलियन डॉलर है, तो अनुमानित औसत बिक्री राजस्व ज्ञात कीजिए। इसके अलावा, 30 मिलियन डॉलर के विज्ञापन व्यय के अनुरूप बिक्री राजस्व के वास्तविक अनुमानित औसत के लिए 99% कॉन्फिडेंस इंटरवल ज्ञात कीजिए।

4. In a regression of housing expenditure in rupees (Y_i) on annual income of families in rupees (X_i) for a sample of 27 families, the following results were obtained :

Variable	Coefficients	Standard Error
X	0.121	0.009
Constant	3.803	4.570
n = 27		
R^2= 0.776		

On plotting the residuals against X_i, it was found that the variance of the residuals increased with X_i

(i) What problem does this indicate? Name any one test for its detection and explain the steps to conduct this test.

(ii) What are the consequences of this problem for OLS estimators? Which type of dataset is more likely to be characterized by this problem?

(iii) Explain the estimation process of Weighted Least Squares with unknown error variances in this context where error variance is proportional to X^2. Also, mention which is the intercept and slope parameter in the transformed model and how do we get back to the original model. (6,6,6)

27 परिवारों के नमूने के लिए (X_i) रुपये की परिवारों की वार्षिक आय में से (Y_i) रुपये के आवास व्यय के प्रतिगमन में, निम्नलिखित परिणाम प्राप्त हुए थे :

चर	गुणांक	मानक त्रुटि
X	0.121	0.009
स्थिरांक	3.803	4.570
$n = 27$		
$R^2 = 0.776$		

X_i के समक्ष अवशिष्टों को रखे जाने पर यह पाया गया कि अवशिष्टों के विचरण में X_i की वृद्धि हुई है।

(i) यह किस समस्या को इंगित करता है? इसकी पहचान के लिए किसी एक परीक्षण का नाम बताइए और इस परीक्षण को करने के चरणों की व्याख्या कीजिए।

(ii) ओएलएस अनुमानक के लिए इस समस्या के परिणाम क्या हैं? इस समस्या से किस प्रकार के आंकड़ा समुच्चय की विशेषता होने की अधिक संभावना है?

(iii) इस संदर्भ में अज्ञात त्रुटि भिन्नताओं के साथ भारित कम से कम वर्गों की अनुमान प्रक्रिया की व्याख्या कीजिए जहां त्रुटि विचरण X^2 के समानुपाती है। इसके अलावा, रूपांतरित मॉडल में अवरोधन और ढलान पैरामीटर कौन-सा होता है और हम मूल मॉडल पर वापस कैसे आते हैं, इसका उल्लेख कीजिए।

5. (a) Suppose a third-degree polynomial regression was fitted to a cost-output model for 18 firms and the following results were obtained :

$$\widehat{Cost}_i = 141.7667 + 0.208\,Q_i - 0.086\,Q_i^2 + 0.0054\,Q_i^3$$
$$se = (6.37) \qquad (4.78) \qquad (2.98) \qquad (6.98)$$

Q_i is the output of the i^{th} firm.

(i) If cost curves are to have U-shaped average and marginal cost curves, then what are your a priori expectations about the intercept and slope estimators? Check if these a priori expectations are satisfied for the given model.

(ii) Interpret the estimated slope coefficient of Q^2 and test its significance at 10% level of significance.

(b) Let Y be the Gross National Product, X_2 be the Exports, X_3 be the Imports and X_4 be the Net Exports in the following relationship:

$$Y_i = B_1 + B_2X_{2i} + B_3X_{3i} + B_4X_{4i} + U_i$$

Which assumptions of Classical Linear Regression Model is violated here? If you estimate this equation by ordinary least squares, then which of the parameters can be estimated? Explain. (6+6,6)

(क) मान लीजिए कि एक तृतीय-डिग्री बहुपद प्रतिगमन को 18 फर्मों के लिए लागत-आउटपुट मॉडल में फिट किया गया था जिससे निम्नलिखित परिणाम प्राप्त किए गए थे :

$$\widehat{Cost}_i = 141.7667 + 0.208\,Q_i - 0.086\,Q_i^2 + 0.0054\,Q_i^3$$
$$se = (6.37) \qquad (4.78) \qquad (2.98) \qquad (6.98)$$

Q_i, i^{th} फर्म का आउटपुट है।

(i) यदि लागत वक्रों में यू-आकार का औसत और सीमांत लागत वक्र होते हैं, तो आप अवरोधन और ढलान अनुमानक के बारे में प्राथमिकता की क्या अपेक्षा कर रहे हैं? क्या दी गई प्राथमिकता अपेक्षाएं दिए गए मॉडल को संतुष्ट करती हैं।

(ii) Q^2 के अनुमानित ढलान गुणांक की व्याख्या कीजिए और महत्ता के 10% स्तर पर इसके महत्ता का परीक्षण कीजिए।

(ख) मान लीजिए Y सकल राष्ट्रीय उत्पाद है, X_2 निर्यात है, आयात X_3 है और निम्नलिखित संबंध में शुद्ध निर्यात X_4 हैं:

$$Y_i = B_1 + B_2X_{2i} + B_3X_{3i} + B_4X_{4i} + U_i$$

यहां शास्त्रीय रेखीय प्रतिगमन मॉडल की कौन-सी मान्यताओं का उल्लंघन किया गया है? यदि आप इस समीकरण का अनुमान साधारण न्यूनतम वर्गों से लगाते हैं, तो किस पैरामीटर का अनुमान लगाया जा सकता है? स्पष्ट कीजिए।

6. (a) For a particular cafeteria, the following equation was estimated using yearly data on cups of chocolate flavoured cold coffee sold :

$$\widehat{\log(Q_t)} = 1.534 - 0.250\, log(P_t) + 0.025 \log(P_t^{**})$$
$$se = (0.2001) \quad (0.240) \qquad (0.016)$$
$$R^2 = 0.804 \qquad TSS = 4803$$

Where

Q_t = cups of chocolate flavoured cold coffee sold

P_t = price (in rupees) per cup of chocolate flavoured cold coffee

P_t^{**} = price (in rupees) per cup of vanilla flavoured cold coffee

t = 1991-2023

Note that all cups in the cafeteria are of same size.

(i) What can you say about the own-price and cross-price elasticity of demand for chocolate flavoured coffee? Test the hypothesis that the own price elasticity of demand of chocolate flavoured coffee is unitary elastic at 10% level of significance.

(ii) Draw the ANOVA table for the regression equation. Also calculate $\overline{R^2}$.

(b) Explain the statement "It is better to include irrelevant variables than to exclude relevant ones". (6+6,6)

(क) एक विशेष कैफेटेरिया के लिए, चॉकलेट फ्लेवर्ड कोल्ड कॉफी के बेचे गए कप से संबंधित वार्षिक डेटा का उपयोग करके निम्नलिखित समीकरण का अनुमान लगाया गया था :

$$\widehat{\log(Q_t)} = 1.534 - 0.250\, log(P_t) + 0.025 \log(P_t^{**})$$

$$se = (0.2001) \quad (0.240) \qquad (0.016)$$

$$R^2 = 0.804 \qquad TSS = 4803$$

जहां

Q_t = चॉकलेट फ्लेवर्ड कोल्ड कॉफी के बेचे गए कप

P_t = प्रति कप चॉकलेट फ्लेवर्ड कोल्ड कॉफी की कीमत (रुपये में)

P_t^{**} = वनीला फ्लेवर्ड कोल्ड कॉफी की प्रति कप कीमत (रुपये में)

t = 1991-2023

नोट कीजिए कि कैफेटेरिया में सभी कप एक ही आकार के हैं।

(i) चॉकलेट फ्लेवर्ड कॉफी की मांग की स्वयं की कीमत और क्रॉस-कीमत लोच के बारे में आप क्या कहेंगे? इस परिकल्पना का परीक्षण कीजिए कि चॉकलेट फ्लेवर्ड कॉफी की माँग की अपनी कीमत लोच महत्ता के 10% स्तर पर एकात्मक लोचदार है।

(ii) प्रतिगमन समीकरण के लिए एनोवा तालिका तैयार कीजिए साथ ही $\overline{R^2}$ की भी गणना कीजिए।

(ख) "प्रासंगिक चर को बाहर करने की तुलना में अप्रासंगिक चर को शामिल करना बेहतर है"। इस कथन की व्याख्या कीजिए।

7. (a) Explain the steps involved in performing Ramsey's RESET test for model specification.

(b) Using data collected from 1990 to 2022, for a particular country, the following equation was estimated using OLS :

$$\hat{Y}_i = 4.47 + 0.34 \log X_{2i} - 1.22 \log X_{3i}$$
$$t = \quad (4.28) \quad (5.31) \qquad (-0.98)$$
$$R^2 = 0.9987$$

Y = Child Mortality Rate

X_2 = Female Literacy Rate

X_3 = Per Capita GNP

(i) Are the regression results in conformity with your a priori expectations? If not, what can be the reason behind it? Explain any three remedies to overcome this problem.

(ii) Now suppose the researcher regressed female literacy rate on per capita GNP and in this second regression, the R^2 is 0.5284. Do you think that the reason that you suggested in (i) is significantly present in the model, at 5% level of significance? If your answer is yes, then do you think that the presence of this is necessarily bad? (6,6+6)

(क) मॉडल विनिर्देश के लिए रैमसे के रीसेट परीक्षण करने में शामिल चरणों की व्याख्या कीजिए।

(ख) किसी विशेष देश के लिए 1990 से 2022 तक एकत्र किए गए डेटा का उपयोग करते हुए, ओएलएस का उपयोग करके निम्नलिखित समीकरण क

$$\hat{Y}_i = 4.47 + 0.34 \log X_{2i} - 1.22 \log X_{3i}$$
$$t = \ (4.28) \ \ (5.31) \qquad (-0.98)$$
$$R^2 = 0.9987$$

Y = बाल मृत्यु दर

X_2 = महिला साक्षरता दर

X_3 = प्रति व्यक्ति सकल राष्ट्रीय उत्पाद (जीएनपी)

(i) क्या प्रतिगमन परिणाम आपकी प्राथमिकता अपेक्षाओं के अनुरूप हैं? यदि नहीं, तो इसके पीछे क्या कारण हो सकता है? इस समस्या को दूर करने के कोई तीन उपाय बताइए।

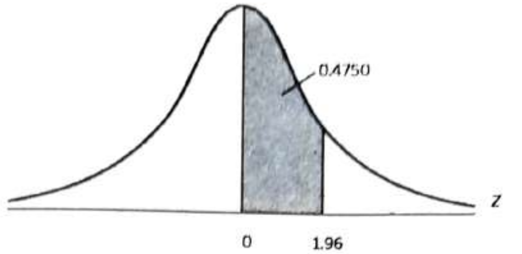
(ii) अब मान लीजिए कि शोधकर्ता ने प्रति व्यक्ति जीएनपी पर महिला साक्षरता दर को छोड़ दिया और इस दूसरे प्रतिगमन में, R^2, 0.5284 है। क्या आपको लगता है कि आपने (i) में जो कारण सुझाया है, वह मॉडल में 5% महत्ता स्तर पर महत्वपूर्ण रूप से मौजूद है? यदि आपका उत्तर हाँ है, तो क्या आपको लगता है कि इसकी उपस्थिति निश्चित रूप से खराब है?

TABLE E-1a AREAS UNDER THE STANDARDIZED NORMAL DISTRIBUTION

Example

$\Pr(0 \leq Z \leq 1.96) = 0.4750$

$\Pr(Z \geq 1.96) = 0.5 - 0.4750 = 0.025$



Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4454	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

Note: This table gives the area in the right-hand tail of the distribution (i.e., *Z* Ú 0). But since the normal distribution is symmetrical about *Z* = 0, the area in the left-hand tail is the same as the area in the corresponding right-hand tail. For example, *P*(-1.96 ... *Z* ... 0) = 0.4750. Therefore, *P*(-1.96 ... *Z* ... 1.96) = 2(0.4750) = 0.95.

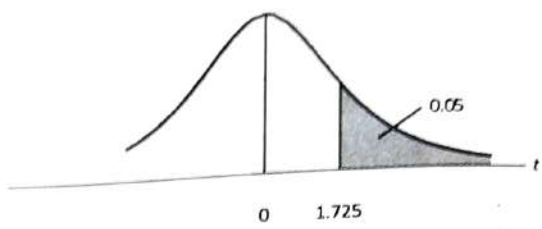
TABLE E-2 PERCENTAGE POINTS OF THE t DISTRIBUTION

Example

$\Pr(t > 2.086) = 0.025$

$\Pr(t > 1.725) = 0.05$ for d.f. = 20

$\Pr(|t| > 1.725) = 0.10$



d.f. \ Pr	0.25 0.50	0.10 0.20	0.05 0.10	0.025 0.05	0.01 0.02	0.005 0.010	0.001 0.002
1	1.000	3.078	6.314	12.706	31.821	63.657	318.31
2	0.816	1.886	2.920	4.303	6.965	9.925	22.327
3	0.765	1.638	2.353	3.182	4.541	5.841	10.214
4	0.741	1.533	2.132	2.776	3.747	4.604	7.173
5	0.727	1.476	2.015	2.571	3.365	4.032	5.893
6	0.718	1.440	1.943	2.447	3.143	3.707	5.208
7	0.711	1.415	1.895	2.365	2.998	3.499	4.785
8	0.706	1.397	1.860	2.306	2.896	3.355	4.501
9	0.703	1.383	1.833	2.262	2.821	3.250	4.297
10	0.700	1.372	1.812	2.228	2.764	3.169	4.144
11	0.697	1.363	1.796	2.201	2.718	3.106	4.025
12	0.695	1.356	1.782	2.179	2.681	3.055	3.930
13	0.694	1.350	1.771	2.160	2.650	3.012	3.852
14	0.692	1.345	1.761	2.145	2.624	2.977	3.787
15	0.691	1.341	1.753	2.131	2.602	2.947	3.733
16	0.690	1.337	1.746	2.120	2.583	2.921	3.686
17	0.689	1.333	1.740	2.110	2.567	2.898	3.646
18	0.688	1.330	1.734	2.101	2.552	2.878	3.610
19	0.688	1.328	1.729	2.093	2.539	2.861	3.579
20	0.687	1.325	1.725	2.086	2.528	2.845	3.552
21	0.686	1.323	1.721	2.080	2.518	2.831	3.527
22	0.686	1.321	1.717	2.074	2.508	2.819	3.505
23	0.685	1.319	1.714	2.069	2.500	2.807	3.485
24	0.685	1.318	1.711	2.064	2.492	2.797	3.467
25	0.684	1.316	1.708	2.060	2.485	2.787	3.450
26	0.684	1.315	1.706	2.056	2.479	2.779	3.435
27	0.684	1.314	1.703	2.052	2.473	2.771	3.421
28	0.683	1.313	1.701	2.048	2.467	2.763	3.408
29	0.683	1.311	1.699	2.045	2.462	2.756	3.396
30	0.683	1.310	1.697	2.042	2.457	2.750	3.385
40	0.681	1.303	1.684	2.021	2.423	2.704	3.307
60	0.679	1.296	1.671	2.000	2.390	2.660	3.232
120	0.677	1.289	1.658	1.980	2.358	2.617	3.160
q	0.674	1.282	1.645	1.960	2.326	2.576	3.090

Note: The smaller probability shown at the head of each column is the area in one tail; the larger probability is the area in both tails.

Source: From E. S. Pearson and H. O. Hartley, eds., *Biometrika Tables for Statisticians*, vol. 1, 3rd ed., Table 12, Cambridge University Press, New York, 1966. Reproduced by permission of the editors and trustees of *Biometrika*.

TABLE E-3 UPPER PERCENTAGE POINTS OF THE F DISTRIBUTION

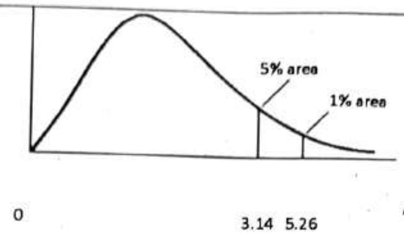
Example

$\Pr(F > 1.59) = 0.25$

$\Pr(F > 2.42) = 0.10$ for d.f. $N_1 = 10$

$\Pr(F > 3.14) = 0.05$ and $N_2 = 9$

$\Pr(F > 5.26) = 0.01$



d.f. for denom-inator N_2		d.f. for numerator N_1											
	Pr	1	2	3	4	5	6	7	8	9	10	11	12
1	.25	5.83	7.50	8.20	8.58	8.82	8.98	9.10	9.19	9.26	9.32	9.36	9.41
	.10	39.90	49.50	53.60	55.80	57.20	58.20	58.90	59.40	59.90	60.20	60.50	60.70
	.05	161.00	200.00	216.00	225.00	230.00	234.00	237.00	239.00	241.00	242.00	243.00	244.00
2	.25	2.57	3.00	3.15	3.23	3.28	3.31	3.34	3.35	3.37	3.38	3.39	3.39
	.10	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38	9.39	9.40	9.41
	.05	18.50	19.00	19.20	19.20	19.30	19.30	19.40	19.40	19.40	19.40	19.40	19.40
	.01	98.50	99.00	99.20	99.20	99.30	99.30	99.40	99.40	99.40	99.40	99.40	99.40
3	.25	2.02	2.28	2.36	2.39	2.41	2.42	2.43	2.44	2.44	2.44	2.45	2.45
	.10	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24	5.23	5.22	5.22
	.05	10.10	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.76	8.74
	.01	34.10	30.80	29.50	28.70	28.20	27.90	27.70	27.50	27.30	27.20	27.10	27.10
4	.25	1.81	2.00	2.05	2.06	2.07	2.08	2.08	2.08	2.08	2.08	2.08	2.08
	.10	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94	3.92	3.91	3.90
	.05	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.94	5.91
	.01	21.20	18.00	16.70	16.00	15.50	15.20	15.00	14.80	14.70	14.50	14.40	14.40
5	.25	1.69	1.85	1.88	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89
	.10	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32	3.30	3.28	3.27
	.05	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.71	4.68
	.01	16.30	13.30	12.10	11.40	11.00	10.70	10.50	10.30	10.20	10.10	9.96	9.89
6	.25	1.62	1.76	1.78	1.79	1.79	1.78	1.78	1.78	1.77	1.77	1.77	1.77
	.10	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96	2.94	2.92	2.90
	.05	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.03	4.00
	.01	13.70	10.90	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.79	7.72
7	.25	1.57	1.70	1.72	1.72	1.71	1.71	1.70	1.70	1.69	1.69	1.69	1.68
	.10	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72	2.70	2.68	2.67
	.05	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.60	3.57
	.01	12.20	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.54	6.47
8	.25	1.54	1.66	1.67	1.66	1.66	1.65	1.64	1.64	1.63	1.63	1.63	1.62
	.10	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56	2.54	2.52	2.50
	.05	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.31	3.28
	.01	11.30	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.73	5.67
9	.25	1.51	1.62	1.63	1.63	1.62	1.61	1.60	1.60	1.59	1.59	1.58	1.58
	.10	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44	2.42	2.40	2.38
	.05	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.10	3.07
	.01	10.60	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.18	5.11

Source: From E. S. Pearson and H. O. Hartley, eds., *Biometrika Tables for Statisticians*, vol. 1, 3rd ed., Table 18, Cambridge University Press, New York, 1966. Reproduced by permission of the editors and trustees of *Biometrika*.

d.f. for numerator N_1													d.f. for denominator
15	20	24	30	40	50	60	100	120	200	500	q	Pr	N_2
9.49	9.58	9.63	9.67	9.71	9.74	9.76	9.78	9.80	9.82	9.84	9.85	.25	
61.20	61.70	62.00	62.30	62.50	62.70	62.80	63.00	63.10	63.20	63.30	63.30	.10	1
246.00	248.00	249.00	250.00	251.00	252.00	252.00	253.00	253.00	254.00	254.00	254.00	.05	
3.41	3.43	3.43	3.44	3.45	3.45	3.46	3.47	3.47	3.48	3.48	3.48	.25	
9.42	9.44	9.45	9.46	9.47	9.47	9.47	9.48	9.48	9.49	9.49	9.49	.10	2
19.40	19.40	19.50	19.50	19.50	19.50	19.50	19.50	19.50	19.50	19.50	19.50	.05	
99.40	99.40	99.50	99.50	99.50	99.50	99.50	99.50	99.50	99.50	99.50	99.50	.01	
2.46	2.46	2.46	2.47	2.47	2.47	2.47	2.47	2.47	2.47	2.47	2.47	.25	
5.20	5.18	5.18	5.17	5.16	5.15	5.15	5.14	5.14	5.14	5.14	5.13	.10	3
8.70	8.66	8.64	8.62	8.59	8.58	8.57	8.55	8.55	8.54	8.53	8.53	.05	
26.90	26.70	26.60	26.50	26.40	26.40	26.30	26.20	26.20	26.20	26.10	26.10	.01	
2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	.25	
3.87	3.84	3.83	3.82	3.80	3.80	3.79	3.78	3.78	3.77	3.76	3.76	.10	4
5.86	5.80	5.77	5.75	5.72	5.70	5.69	5.66	5.66	5.65	5.64	5.63	.05	
14.20	14.00	13.90	13.80	13.70	13.70	13.70	13.60	13.60	13.50	13.50	13.50	.01	
1.89	1.88	1.88	1.88	1.88	1.88	1.87	1.87	1.87	1.87	1.87	1.87	.25	
3.24	3.21	3.19	3.17	3.16	3.15	3.14	3.13	3.12	3.12	3.11	3.10	.10	5
4.62	4.56	4.53	4.50	4.46	4.44	4.43	4.41	4.40	4.39	4.37	4.36	.05	
9.72	9.55	9.47	9.38	9.29	9.24	9.20	9.13	9.11	9.08	9.04	9.02	.01	
1.76	1.76	1.75	1.75	1.75	1.75	1.74	1.74	1.74	1.74	1.74	1.74	.25	
2.87	2.84	2.82	2.80	2.78	2.77	2.76	2.75	2.74	2.73	2.73	2.72	.10	6
3.94	3.87	3.84	3.81	3.77	3.75	3.74	3.71	3.70	3.69	3.68	3.67	.05	
7.56	7.40	7.31	7.23	7.14	7.09	7.06	6.99	6.97	6.93	6.90	6.88	.01	
1.68	1.67	1.67	1.66	1.66	1.66	1.65	1.65	1.65	1.65	1.65	1.65	.25	
2.63	2.59	2.58	2.56	2.54	2.52	2.51	2.50	2.49	2.48	2.48	2.47	.10	7
3.51	3.44	3.41	3.38	3.34	3.32	3.30	3.27	3.27	3.25	3.24	3.23	.05	
6.31	6.16	6.07	5.99	5.91	5.86	5.82	5.75	5.74	5.70	5.67	5.65	.01	
1.62	1.61	1.60	1.60	1.59	1.59	1.59	1.58	1.58	1.58	1.58	1.58	.25	
2.46	2.42	2.40	2.38	2.36	2.35	2.34	2.32	2.32	2.31	2.30	2.29	.10	8
3.22	3.15	3.12	3.08	3.04	2.02	3.01	2.97	2.97	2.95	2.94	2.93	.05	
5.52	5.36	5.28	5.20	5.12	5.07	5.03	4.96	4.95	4.91	4.88	4.86	.01	
1.57	1.56	1.56	1.55	1.55	1.54	1.54	1.53	1.53	1.53	1.53	1.53	.25	
2.34	2.30	2.28	2.25	2.23	2.22	2.21	2.19	2.18	2.17	2.17	2.16	.10	9
3.01	2.94	2.90	2.86	2.83	2.80	2.79	2.76	2.75	2.73	2.72	2.71	.05	
4.96	4.81	4.73	4.65	4.57	4.52	4.48	4.42	4.40	4.36	4.33	4.31	.01	

TABLE E-3 UPPER PERCENTAGE POINTS OF THE F DISTRIBUTION (CONTINUED)

d.f. for denominator N_2	d.f. for numerator N_1												
	Pr	1	2	3	4	5	6	7	8	9	10	11	12
10	.25	1.49	1.60	1.60	1.59	1.59	1.58	1.57	1.56	1.56	1.55	1.55	1.54
	.10	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35	2.32	2.30	2.28
	.05	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.94	2.91
	.01	10.00	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.77	4.71
11	.25	1.47	1.58	1.58	1.57	1.56	1.55	1.54	1.53	1.53	1.52	1.52	1.51
	.10	3.23	2.86	2.66	2.54	2.45	2.39	2.34	2.30	2.27	2.25	2.23	2.21
	.05	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.82	2.79
	.01	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.46	4.40
12	.25	1.46	1.56	1.56	1.55	1.54	1.53	1.52	1.51	1.51	1.50	1.50	1.49
	.10	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21	2.19	2.17	2.15
	.05	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.72	2.69
	.01	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.22	4.16
13	.25	1.45	1.55	1.55	1.53	1.52	1.51	1.50	1.49	1.49	1.48	1.47	1.47
	.10	3.14	2.76	2.56	2.43	2.35	2.28	2.23	2.20	2.16	2.14	2.12	2.10
	.05	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.63	2.60
	.01	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	4.02	3.96
14	.25	1.44	1.53	1.53	1.52	1.51	1.50	1.49	1.48	1.47	1.46	1.46	1.45
	.10	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.12	2.10	2.08	2.05
	.05	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.57	2.53
	.01	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94	3.86	3.80
15	.25	1.43	1.52	1.52	1.51	1.49	1.48	1.47	1.46	1.46	1.45	1.44	1.44
	.10	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09	2.06	2.04	2.02
	.05	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.51	2.48
	.01	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.73	3.67
16	.25	1.42	1.51	1.51	1.50	1.48	1.47	1.46	1.45	1.44	1.44	1.44	1.43
	.10	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.06	2.03	2.01	1.99
	.05	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.46	2.42
	.01	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.62	3.55
17	.25	1.42	1.51	1.50	1.49	1.47	1.46	1.45	1.44	1.43	1.43	1.42	1.41
	.10	3.03	2.64	2.44	2.31	2.22	2.15	2.10	2.06	2.03	2.00	1.98	1.96
	.05	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.41	2.38
	.01	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.52	3.46
18	.25	1.41	1.50	1.49	1.48	1.46	1.45	1.44	1.43	1.42	1.42	1.41	1.40
	.10	3.01	2.62	2.42	2.29	2.20	2.13	2.08	2.04	2.00	1.98	1.96	1.93
	.05	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.37	2.34
	.01	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.43	3.37
19	.25	1.41	1.49	1.49	1.47	1.46	1.44	1.43	1.42	1.41	1.41	1.40	1.40
	.10	2.99	2.61	2.40	2.27	2.18	2.11	2.06	2.02	1.98	1.96	1.94	1.91
	.05	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.34	2.31
	.01	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.36	3.30
20	.25	1.40	1.49	1.48	1.46	1.45	1.44	1.43	1.42	1.41	1.40	1.39	1.39
	.10	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96	1.94	1.92	1.89
	.05	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.31	2.28
	.01	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.29	3.23

d.f. for numerator N_1													d.f. for denominator N_2
15	20	24	30	40	50	60	100	120	200	500	∞	Pr	
1.53	1.52	1.52	1.51	1.51	1.50	1.50	1.49	1.49	1.49	1.48	1.48	.25	10
2.24	2.20	2.18	2.16	2.13	2.12	2.11	2.09	2.08	2.07	2.06	2.06	.10	
2.85	2.77	2.74	2.70	2.66	2.64	2.62	2.59	2.58	2.56	2.55	2.54	.05	
4.56	4.41	4.33	4.25	4.17	4.12	4.08	4.01	4.00	3.96	3.93	3.91	.01	
1.50	1.49	1.49	1.48	1.47	1.47	1.47	1.46	1.46	1.46	1.45	1.45	.25	11
2.17	2.12	2.10	2.08	2.05	2.04	2.03	2.00	2.00	1.99	1.98	1.97	.10	
2.72	2.65	2.61	2.57	2.53	2.51	2.49	2.46	2.45	2.43	2.42	2.40	.05	
4.25	4.10	4.02	3.94	3.86	3.81	3.78	3.71	3.69	3.66	3.62	3.60	.01	
1.48	1.47	1.46	1.45	1.45	1.44	1.44	1.43	1.43	1.43	1.42	1.42	.25	12
2.10	2.06	2.04	2.01	1.99	1.97	1.96	1.94	1.93	1.92	1.91	1.90	.10	
2.62	2.54	2.51	2.47	2.43	2.40	2.38	2.35	2.34	2.32	2.31	2.30	.05	
4.01	3.86	3.78	3.70	3.62	3.57	3.54	3.47	3.45	3.41	3.38	3.36	.01	
1.46	1.45	1.44	1.43	1.42	1.42	1.42	1.41	1.41	1.40	1.40	1.40	.25	13
2.05	2.01	1.98	1.96	1.93	1.92	1.90	1.88	1.88	1.86	1.85	1.85	.10	
2.53	2.46	2.42	2.38	2.34	2.31	2.30	2.26	2.25	2.23	2.22	2.21	.05	
3.82	3.66	3.59	3.51	3.43	3.38	3.34	3.27	3.25	3.22	3.19	3.17	.01	
1.44	1.43	1.42	1.41	1.41	1.40	1.40	1.39	1.39	1.39	1.38	1.38	.25	14
2.01	1.96	1.94	1.91	1.89	1.87	1.86	1.83	1.83	1.82	1.80	1.80	.10	
2.46	2.39	2.35	2.31	2.27	2.24	2.22	2.19	2.18	2.16	2.14	2.13	.05	
3.66	3.51	3.43	3.35	3.27	3.22	3.18	3.11	3.09	3.06	3.03	3.00	.01	
1.43	1.41	1.41	1.40	1.39	1.39	1.38	1.38	1.37	1.37	1.36	1.36	.25	15
1.97	1.92	1.90	1.87	1.85	1.83	1.82	1.79	1.79	1.77	1.76	1.76	.10	
2.40	2.33	2.29	2.25	2.20	2.18	2.16	2.12	2.11	2.10	2.08	2.07	.05	
3.52	3.37	3.29	3.21	3.13	3.08	3.05	2.98	2.96	2.92	2.89	2.87	.01	
1.41	1.40	1.39	1.38	1.37	1.37	1.36	1.36	1.35	1.35	1.34	1.34	.25	16
1.94	1.89	1.87	1.84	1.81	1.79	1.78	1.76	1.75	1.74	1.73	1.72	.10	
2.35	2.28	2.24	2.19	2.15	2.12	2.11	2.07	2.06	2.04	2.02	2.01	.05	
3.41	3.26	3.18	3.10	3.02	2.97	2.93	2.86	2.84	2.81	2.78	2.75	.01	
1.40	1.39	1.38	1.37	1.36	1.35	1.35	1.34	1.34	1.34	1.33	1.33	.25	17
1.91	1.86	1.84	1.81	1.78	1.76	1.75	1.73	1.72	1.71	1.69	1.69	.10	
2.31	2.23	2.19	2.15	2.10	2.08	2.06	2.02	2.01	1.99	1.97	1.96	.05	
3.31	3.16	3.08	3.00	2.92	2.87	2.83	2.76	2.75	2.71	2.68	2.65	.01	
1.39	1.38	1.37	1.36	1.35	1.34	1.34	1.33	1.33	1.32	1.32	1.32	.25	18
1.89	1.84	1.81	1.78	1.75	1.74	1.72	1.70	1.69	1.68	1.67	1.66	.10	
2.27	2.19	2.15	2.11	2.06	2.04	2.02	1.98	1.97	1.95	1.93	1.92	.05	
3.23	3.08	3.00	2.92	2.84	2.78	2.75	2.68	2.66	2.62	2.59	2.57	.01	
1.38	1.37	1.36	1.35	1.34	1.33	1.33	1.32	1.32	1.31	1.31	1.30	.25	19
1.86	1.81	1.79	1.76	1.73	1.71	1.70	1.67	1.67	1.65	1.64	1.63	.10	
2.23	2.16	2.11	2.07	2.03	2.00	1.98	1.94	1.93	1.91	1.89	1.88	.05	
3.15	3.00	2.92	2.84	2.76	2.71	2.67	2.60	2.58	2.55	2.51	2.49	.01	
1.37	1.36	1.35	1.34	1.33	1.33	1.32	1.31	1.31	1.30	1.30	1.29	.25	20
1.84	1.79	1.77	1.74	1.71	1.69	1.68	1.65	1.64	1.63	1.62	1.61	.10	
2.20	2.12	2.08	2.04	1.99	1.97	1.95	1.91	1.90	1.88	1.86	1.84	.05	
3.09	2.94	2.86	2.78	2.69	2.64	2.61	2.54	2.52	2.48	2.44	2.42	.01	

TABLE E-3 UPPER PERCENTAGE POINTS OF THE *F* DISTRIBUTION (CONTINUED)

d.f. for denom- inator N_2		d.f. for numerator N_1											
	Pr	1	2	3	4	5	6	7	8	9	10	11	12
	.25	1.40	1.48	1.47	1.45	1.44	1.42	1.41	1.40	1.39	1.39	1.38	1.37
22	.10	2.95	2.56	2.35	2.22	2.13	2.06	2.01	1.97	1.93	1.90	1.88	1.86
	.05	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.26	2.23
	.01	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.18	3.12
	.25	1.39	1.47	1.46	1.44	1.43	1.41	1.40	1.39	1.38	1.38	1.37	1.36
24	.10	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91	1.88	1.85	1.83
	.05	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.21	2.18
	.01	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.09	3.03
	.25	1.38	1.46	1.45	1.44	1.42	1.41	1.39	1.38	1.37	1.37	1.36	1.35
26	.10	2.91	2.52	2.31	2.17	2.08	2.01	1.96	1.92	1.88	1.86	1.84	1.81
	.05	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.18	2.15
	.01	7.72	5.53	4.64	4.14	3.82	3.59	3,42	3.29	3.18	3.09	3.02	2.96
	.25	1.38	1.46	1.45	1.43	1.41	1.40	1.39	1.38	1.37	1.36	1.35	1.34
28	.10	2.89	2.50	2.29	2.16	2.06	2.00	1.94	1.90	1.87	1.84	1.81	1.79
	.05	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.15	2.12
	.01	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.96	2.90
	.25	1.38	1.45	1.44	1.42	1.41	1.39	1.38	1.37	1.36	1.35	1.35	1.34
30	.10	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85	1.82	1.79	1.77
	.05	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.13	2.09
	.01	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.91	2.84
	.25	1.36	1.44	1.42	1.40	1.39	1.37	1.36	1.35	1.34	1.33	1.32	1.31
40	.10	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79	1.76	1.73	1.71
	.05	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.04	2.00
	.01	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.73	2.66
	.25	1.35	1.42	1.41	1.38	1.37	1.35	1.33	1.32	1.31	1.30	1.29	1.29
60	.10	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74	1.71	1.68	1.66
	.05	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.95	1.92
	.01	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.56	2.50
	.25	1.34	1.40	1.39	1.37	1.35	1.33	1.31	1.30	1.29	1.28	1.27	1.26
120	.10	2.75	2.35	2.13	1,99	1.90	1.82	1.77	1.72	1.68	1.65	1.62	1.60
	.05	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.87	1.83
	.01	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.40	2.34
	.25	1.33	1.39	1.38	1.36	1.34	1.32	1.31	1.29	1.28	1.27	1.26	1.25
200	.10	2.73	2.33	2.11	1.97	1.88	1.80	1.75	1.70	1.66	1.63	1.60	1.57
	.05	3.89	3.04	2.65	2.42	2.26	2.14	2.06	1.98	1.93	1.88	1.84	1.80
	.01	6.76	4.71	3.88	3.41	3.11	2.89	2.73	2.60	2.50	2.41	2.34	2.27
	.25	1.32	1.39	1.37	1.35	1.33	1.31	1.29	1.28	1.27	1.25	1.24	1.24
q	.10	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63	1.60	1.57	1.55
	.05	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.79	1.75
	.01	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.25	2.18

P.T.O.

d.f. for numerator N_1													d.f. for denominator N_2
15	20	24	30	40	50	60	100	120	200	500	∞	Pr	
1.36	1.34	1.33	1.32	1.31	1.31	1.30	1.30	1.30	1.29	1.29	1.28	.25	22
1.81	1.76	1.73	1.70	1.67	1.65	1.64	1.61	1.60	1.59	1.58	1.57	.10	
2.15	2.07	2.03	1.98	1.94	1.91	1.89	1.85	1.84	1.82	1.80	1.78	.05	
2.98	2.83	2.75	2.67	2.58	2.53	2.50	2.42	2.40	2.36	2.33	2.31	.01	
1.35	1.33	1.32	1.31	1.30	1.29	1.29	1.28	1.28	1.27	1.27	1.26	.25	24
1.78	1.73	1.70	1.67	1.64	1.62	1.61	1.58	1.57	1.56	1.54	1.53	.10	
2.11	2.03	1.98	1.94	1.89	1.86	1.84	1.80	1.79	1.77	1.75	1.73	.05	
2.89	2.74	2.66	2.58	2.49	2.44	2.40	2.33	2.31	2.27	2.24	2.21	.01	
1.34	1.32	1.31	1.30	1.29	1.28	1.28	1.26	1.26	1.26	1.25	1.25	.25	26
1.76	1.71	1.68	1.65	1.61	1.59	1.58	1.55	1.54	1.53	1.51	1.50	.10	
2.07	1.99	1.95	1.90	1.85	1.82	1.80	1.76	1.75	1.73	1.71	1.69	.05	
2.81	2.66	2.58	2.50	2.42	2.36	2.33	2.25	2.23	2.19	2.16	2.13	.01	
1.33	1.31	1.30	1.29	1.28	1.27	1.27	1.26	1.25	1.25	1.24	1.24	.25	28
1.74	1.69	1.66	1.63	1.59	1.57	1.56	1.53	1.52	1.50	1.49	1.48	.10	
2.04	1.96	1.91	1.87	1.82	1.79	1.77	1.73	1.71	1.69	1.67	1.65	.05	
2.75	2.60	2.52	2.44	2.35	2.30	2.26	2.19	2.17	2.13	2.09	2.06	.01	
1.32	1.30	1.29	1.28	1.27	1.26	1.26	1.25	1.24	1.24	1.23	1.23	.25	30
1.72	1.67	1.64	1.61	1.57	1.55	1.54	1.51	1.50	1.48	1.47	1.46	.10	
2.01	1.93	1.89	1.84	1.79	1.76	1.74	1.70	1.68	1.66	1.64	1.62	.05	
2.70	2.55	2.47	2.39	2.30	2.25	2.21	2.13	2.11	2.07	2.03	2.01	.01	
1.30	1.28	1.26	1.25	1.24	1.23	1.22	1.21	1.21	1.20	1.19	1.19	.25	40
1.66	1.61	1.57	1.54	1.51	1.48	1.47	1.43	1.42	1.41	1.39	1.38	.10	
1.92	1.84	1.79	1.74	1.69	1.66	1.64	1.59	1.58	1.55	1.53	1.51	.05	
2.52	2.37	2.29	2.20	2.11	2.06	2.02	1.94	1.92	1.87	1.83	1.80	.01	
1.27	1.25	1.24	1.22	1.21	1.20	1.19	1.17	1.17	1.16	1.15	1.15	.25	60
1.60	1.54	1.51	1.48	1.44	1.41	1.40	1.36	1.35	1.33	1.31	1.29	.10	
1.84	1.75	1.70	1.65	1.59	1.56	1.53	1.48	1.47	1.44	1.41	1.39	.05	
2.35	2.20	2.12	2.03	1.94	1.88	1.84	1.75	1.73	1.68	1.63	1.60	.01	
1.24	1.22	1.21	1.19	1.18	1.17	1.16	1.14	1.13	1.12	1.11	1.10	.25	120
1.55	1.48	1.45	1.41	1.37	1.34	1.32	1.27	1.26	1.24	1.21	1.19	.10	
1.75	1.66	1.61	1.55	1.50	1.46	1.43	1.37	1.35	1.32	1.28	1.25	.05	
2.19	2.03	1.95	1.86	1.76	1.70	1.66	1.56	1.53	1.48	1.42	1.38	.01	
1.23	1.21	1.20	1.18	1.16	1.14	1.12	1.11	1.10	1.09	1.08	1.06	.25	200
1.52	1.46	1.42	1.38	1.34	1.31	1.28	1.24	1.22	1.20	1.17	1.14	.10	
1.72	1.62	1.57	1.52	1.46	1.41	1.39	1.32	1.29	1.26	1.22	1.19	.05	
2.13	1.97	1.89	1.79	1.69	1.63	1.58	1.48	1.44	1.39	1.33	1.28	.01	
1.22	1.19	1.18	1.16	1.14	1.13	1.12	1.09	1.08	1.07	1.04	1.00	.25	∞
1.49	1.42	1.38	1.34	1.30	1.26	1.24	1.18	1.17	1.13	1.08	1.00	.10	
1.67	1.57	1.52	1.46	1.39	1.35	1.32	1.24	1.22	1.17	1.11	1.00	.05	
2.04	1.88	1.79	1.70	1.59	1.52	1.47	1.36	1.32	1.25	1.15	1.00	.01	

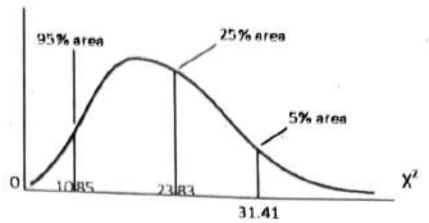
TABLE E-4 UPPER PERCENTAGE POINTS OF THE χ^2 DISTRIBUTION

Example

$\Pr(\chi^2 > 10.85) = 0.95$

$\Pr(\chi^2 > 23.83) = 0.25$ for d.f. = 20

$\Pr(\chi^2 > 31.41) = 0.05$



Degrees of Freedom \ Pr	.995	.990	.975	.950	.900
1	392704×10^{-10}	157088×10^{-9}	982069×10^{-9}	393214×10^{-8}	.0158
2	.0100	.0201	.0506	.1026	.2107
3	.0717	.1148	.2158	.3518	.5844
4	.2070	.2971	.4844	.7107	1.0636
5	.4117	.5543	.8312	1.1455	1.6103
6	.6757	.8721	1.2373	1.6354	2.2041
7	.9893	1.2390	1.6899	2.1674	2.8331
8	1.3444	1.6465	2.1797	2.7326	3.4895
9	1.7349	2.0879	2.7004	3.3251	4.1682
10	2.1559	2.5582	3.2470	3.9403	4.8652
11	2.6032	3.0535	3.8158	4.5748	5.5778
12	3.0738	3.5706	4.4038	5.2260	6.3038
13	3.5650	4.1069	5.0087	5.8919	7.0415
14	4.0747	4.6604	5.6287	6.5706	7.7895
15	4.6009	5.2294	6.2621	7.2609	8.5468
16	5.1422	5.8122	6.9077	7.9616	9.3122
17	5.6972	6.4078	7.5642	8.6718	10.0852
18	6.2648	7.0149	8.2308	9.3905	10.8649
19	6.8440	7.6327	8.9066	10.1170	11.6509
20	7.4339	8.2604	9.5908	10.8508	12.4426
21	8.0337	8.8972	10.2829	11.5913	13.2396
22	8.6427	9.5425	10.9823	12.3380	14.0415
23	9.2604	10.1957	11.6885	13.0905	14.8479
24	9.8862	10.8564	12.4011	13.8484	15.6587
25	10.5197	11.5240	13.1197	14.6114	16.4734
26	11.1603	12.1981	13.8439	15.3791	17.2919
27	11.8076	12.8786	14.5733	16.1513	18.1138
28	12.4613	13.5648	15.3079	16.9279	18.9392
29	13.1211	14.2565	16.0471	17.7083	19.7677
30	13.7867	14.9535	16.7908	18.4926	20.5992
40	20.7065	22.1643	24.4331	26.5093	29.0505
50	27.9907	29.7067	32.3574	34.7642	37.6886
60	35.5346	37.4848	40.4817	43.1879	46.4589
70	43.2752	45.4418	48.7576	51.7393	55.3290
80	51.1720	53.5400	57.1532	60.3915	64.2778
90	59.1963	61.7541	65.6466	69.1260	73.2912
100*	67.3276	70.0648	74.2219	77.9295	82.3581

*For d.f. greater than 100 the expression $\sqrt{2x^2} - \sqrt{(2k-1)} = Z$ follows the standardized normal distribution, where k represents the degrees of freedom.

.750	.500	.250	.100	.050	.025	.010	.005
.1015	.4549	1.3233	2.7055	3.8415	5.0239	6.6349	7.8794
.5754	1.3863	2.7726	4.6052	5.9915	7.3778	9.2103	10.5966
1.2125	2.3660	4.1084	6.2514	7.8147	9.3484	11.3449	12.8381
1.9226	3.3567	5.3853	7.7794	9.4877	11.1433	13.2767	14.8602
2.6746	4.3515	6.6257	9.2364	11.0705	12.8325	15.0863	16.7496
3.4546	5.3481	7.8408	10.6446	12.5916	14.4494	16.8119	18.5476
4.2549	6.3458	9.0372	12.0170	14.0671	16.0128	18.4753	20.2777
5.0706	7.3441	10.2188	13.3616	15.5073	17.5346	20.0902	21.9550
5.8988	8.3428	11.3887	14.6837	16.9190	19.0228	21.6660	23.5893
6.7372	9.3418	12.5489	15.9871	18.3070	20.4831	23.2093	25.1882
7.5841	10.3410	13.7007	17.2750	19.6751	21.9200	24.7250	26.7569
8.4384	11.3403	14.8454	18.5494	21.0261	23.3367	26.2170	28.2995
9.2991	12.3398	15.9839	19.8119	22.3621	24.7356	27.6883	29.8194
10.1653	13.3393	17.1170	21.0642	23.6848	26.1190	29.1413	31.3193
11.0365	14.3389	18.2451	22.3072	24.9958	27.4884	30.5779	32.8013
11.9122	15.3385	19.3688	23.5418	26.2962	28.8454	31.9999	34.2672
12.7919	16.3381	20.4887	24.7690	27.5871	30.1910	33.4087	35.7185
13.6753	17.3379	21.6049	25.9894	28.8693	31.5264	34.8053	37.1564
14.5620	18.3376	22.7178	27.2036	30.1435	32.8523	36.1908	38.5822
15.4518	19.3374	23.8277	28.4120	31.4104	34.1696	37.5662	39.9968
16.3444	20.3372	24.9348	29.6151	32.6705	35.4789	38.9321	41.4010
17.2396	21.3370	26.0393	30.8133	33.9244	36.7807	40.2894	42.7956
18.1373	22.3369	27.1413	32.0069	35.1725	38.0757	41.6384	44.1813
19.0372	23.3367	28.2412	33.1963	36.4151	39.3641	42.9798	45.5585
19.9393	24.3366	29.3389	34.3816	37.6525	40.6465	44.3141	46.9278
20.8434	25.3364	30.4345	35.5631	38.8852	41.9232	45.6417	48.2899
21.7494	26.3363	31.5284	36.7412	40.1133	43.1944	46.9630	49.6449
22.6572	27.3363	32.6205	37.9159	41.3372	44.4607	48.2782	50.9933
23.5666	28.3362	33.7109	39.0875	42.5569	45.7222	49.5879	52.3356
24.4776	29.3360	34.7998	40.2560	43.7729	46.9792	50.8922	53.6720
33.6603	39.3354	45.6160	51.8050	55.7585	59.3417	63.6907	66.7659
42.9421	49.3349	56.3336	63.1671	67.5048	71.4202	76.1539	79.4900
52.2938	59.3347	66.9814	74.3970	79.0819	83.2976	88.3794	91.9517
61.6983	69.3344	77.5766	85.5271	90.5312	95.0231	100.425	104.215
71.1445	79.3343	88.1303	96.5782	101.879	106.629	112.329	116.321
80.6247	89.3342	98.6499	107.565	113.145	118.136	124.116	128.299
90.1332	99.3341	109.141	118.498	124.342	129.561	135.807	140.169

Source: Abridged from E. S. Pearson and H. O. Hartley, eds., *Biometrika Tables for Statisticians*, vol. 1, 3rd ed., Table 8, Cambridge University Press, New York, 1966. Reproduced by permission of the editors and trustees of *Biometrika*.

TABLE E-5a DURBIN-WATSON d STATISTIC: SIGNIFICANCE POINTS OF d_L AND d_U AT 0.05 LEVEL OF SIGNIFICANCE

	$k'=1$		$k'=2$		$k'=3$		$k'=4$		$k'=5$		$k'=6$		$k'=7$		$k'=8$		$k'=9$		$k'=10$	
n	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U
6	0.610	1.400	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
7	0.700	1.356	0.467	1.896	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
8	0.763	1.332	0.559	1.777	0.368	2.287	—	—	—	—	—	—	—	—	—	—	—	—	—	—
9	0.824	1.320	0.629	1.699	0.455	2.128	0.296	2.588	—	—	—	—	—	—	—	—	—	—	—	—
10	0.879	1.320	0.697	1.641	0.525	2.016	0.376	2.414	0.243	2.822	—	—	—	—	—	—	—	—	—	—
11	0.927	1.324	0.658	1.604	0.595	1.928	0.444	2.283	0.316	2.645	0.203	3.005	—	—	—	—	—	—	—	—
12	0.971	1.331	0.812	1.579	0.658	1.864	0.512	2.177	0.379	2.506	0.268	2.832	0.171	3.149	—	—	—	—	—	—
13	1.010	1.340	0.861	1.562	0.715	1.816	0.574	2.094	0.445	2.390	0.328	2.692	0.230	2.985	0.147	3.266	—	—	—	—
14	1.045	1.350	0.905	1.551	0.767	1.779	0.632	2.030	0.505	2.296	0.389	2.572	0.286	2.848	0.200	3.111	0.127	3.360	—	—
15	1.077	1.361	0.946	1.543	0.814	1.750	0.685	1.977	0.562	2.220	0.447	2.472	0.343	2.727	0.251	2.979	0.175	3.216	0.111	3.438
16	1.106	1.371	0.982	1.539	0.857	1.728	0.734	1.935	0.615	2.157	0.502	2.388	0.398	2.624	0.304	2.860	0.222	3.090	0.155	3.304
17	1.133	1.381	1.015	1.536	0.897	1.710	0.779	1.900	0.664	2.104	0.554	2.318	0.451	2.537	0.356	2.757	0.272	2.975	0.198	3.184
18	1.158	1.391	1.046	1.535	0.933	1.696	0.820	1.872	0.710	2.060	0.603	2.257	0.502	2.461	0.407	2.667	0.321	2.873	0.244	3.073
19	1.180	1.401	1.074	1.536	0.967	1.685	0.859	1.848	0.752	2.023	0.649	2.206	0.549	2.396	0.456	2.589	0.369	2.783	0.290	2.974
20	1.201	1.411	1.100	1.537	0.998	1.676	0.894	1.828	0.792	1.991	0.692	2.162	0.595	2.339	0.502	2.521	0.416	2.704	0.336	2.885
21	1.221	1.420	1.125	1.538	1.026	1.669	0.927	1.812	0.829	1.964	0.732	2.124	0.637	2.290	0.547	2.460	0.461	2.633	0.380	2.806
22	1.239	1.429	1.147	1.541	1.053	1.664	0.958	1.797	0.863	1.940	0.769	2.090	0.677	2.246	0.588	2.407	0.504	2.571	0.424	2.734
23	1.257	1.437	1.168	1.543	1.078	1.660	0.986	1.785	0.895	1.920	0.804	2.061	0.715	2.208	0.628	2.360	0.545	2.514	0.465	2.670
24	1.273	1.446	1.188	1.546	1.101	1.656	1.013	1.775	0.925	1.902	0.837	2.035	0.751	2.174	0.666	2.318	0.584	2.464	0.506	2.613
25	1.288	1.454	1.206	1.550	1.123	1.654	1.038	1.767	0.953	1.886	0.868	2.012	0.784	2.144	0.702	2.280	0.621	2.419	0.544	2.560
26	1.302	1.461	1.224	1.553	1.143	1.652	1.062	1.759	0.979	1.873	0.897	1.992	0.816	2.117	0.735	2.246	0.657	2.379	0.581	2.513
27	1.316	1.469	1.240	1.556	1.162	1.651	1.084	1.753	1.004	1.861	0.925	1.974	0.845	2.093	0.767	2.216	0.691	2.342	0.616	2.470
28	1.328	1.476	1.255	1.560	1.181	1.650	1.104	1.747	1.028	1.850	0.951	1.958	0.874	2.071	0.798	2.188	0.723	2.309	0.650	2.431
29	1.341	1.483	1.270	1.563	1.198	1.650	1.124	1.743	1.050	1.841	0.975	1.944	0.900	2.052	0.826	2.164	0.753	2.278	0.682	2.396
30	1.352	1.489	1.284	1.567	1.214	1.650	1.143	1.739	1.071	1.833	0.998	1.931	0.926	2.034	0.854	2.141	0.782	2.251	0.712	2.363
31	1.363	1.496	1.297	1.570	1.229	1.650	1.160	1.735	1.090	1.825	1.020	1.920	0.950	2.018	0.879	2.120	0.810	2.226	0.741	2.333
32	1.373	1.502	1.309	1.574	1.244	1.650	1.177	1.732	1.109	1.819	1.041	1.909	0.972	2.004	0.904	2.102	0.836	2.203	0.769	2.306
33	1.383	1.508	1.321	1.577	1.258	1.651	1.193	1.730	1.127	1.813	1.061	1.900	0.994	1.991	0.927	2.085	0.861	2.181	0.795	2.281
34	1.393	1.514	1.333	1.580	1.271	1.652	1.208	1.728	1.144	1.808	1.080	1.891	1.015	1.979	0.950	2.069	0.885	2.162	0.821	2.257
35	1.402	1.519	1.343	1.584	1.283	1.653	1.222	1.726	1.160	1.803	1.097	1.884	1.034	1.967	0.971	2.054	0.908	2.144	0.845	2.236
36	1.411	1.525	1.354	1.587	1.295	1.654	1.236	1.724	1.175	1.799	1.114	1.877	1.053	1.957	0.991	2.041	0.930	2.127	0.868	2.216
37	1.419	1.530	1.364	1.590	1.307	1.655	1.249	1.723	1.190	1.795	1.131	1.870	1.071	1.948	1.011	2.029	0.951	2.112	0.891	2.198
38	1.427	1.535	1.373	1.594	1.318	1.656	1.261	1.722	1.204	1.792	1.146	1.864	1.088	1.939	1.029	2.017	0.970	2.098	0.912	2.180
39	1.435	1.540	1.382	1.597	1.328	1.658	1.273	1.722	1.218	1.789	1.161	1.859	1.104	1.932	1.047	2.007	0.990	2.085	0.932	2.164
40	1.442	1.544	1.391	1.600	1.338	1.659	1.285	1.721	1.230	1.786	1.175	1.854	1.120	1.924	1.064	1.997	1.008	2.072	0.952	2.149
45	1.475	1.566	1.430	1.615	1.383	1.666	1.336	1.720	1.287	1.776	1.238	1.835	1.189	1.895	1.139	1.958	1.089	2.022	1.038	2.088
50	1.503	1.585	1.462	1.628	1.421	1.674	1.378	1.721	1.335	1.771	1.291	1.822	1.246	1.875	1.201	1.930	1.156	1.986	1.110	2.044
55	1.528	1.601	1.490	1.641	1.452	1.681	1.414	1.724	1.374	1.768	1.334	1.814	1.294	1.861	1.253	1.909	1.212	1.959	1.170	2.010
60	1.549	1.616	1.514	1.652	1.480	1.689	1.444	1.727	1.408	1.767	1.372	1.808	1.335	1.850	1.298	1.894	1.260	1.939	1.222	1.984
65	1.567	1.629	1.536	1.662	1.503	1.696	1.471	1.731	1.438	1.767	1.404	1.805	1.370	1.843	1.336	1.882	1.301	1.923	1.266	1.964
70	1.583	1.641	1.554	1.672	1.525	1.703	1.494	1.735	1.464	1.768	1.433	1.802	1.401	1.837	1.369	1.873	1.337	1.910	1.305	1.948
75	1.598	1.652	1.571	1.680	1.543	1.709	1.515	1.739	1.487	1.770	1.458	1.801	1.428	1.834	1.399	1.867	1.369	1.901	1.339	1.935
80	1.611	1.662	1.586	1.688	1.560	1.715	1.534	1.743	1.507	1.772	1.480	1.801	1.453	1.831	1.425	1.861	1.397	1.893	1.369	1.925
85	1.624	1.671	1.600	1.696	1.575	1.721	1.550	1.747	1.525	1.774	1.500	1.801	1.474	1.829	1.448	1.857	1.422	1.886	1.396	1.916
90	1.635	1.679	1.612	1.703	1.589	1.726	1.566	1.751	1.542	1.776	1.518	1.801	1.494	1.827	1.469	1.854	1.445	1.881	1.420	1.909
95	1.645	1.687	1.623	1.709	1.602	1.732	1.579	1.755	1.557	1.778	1.535	1.802	1.512	1.827	1.489	1.852	1.465	1.877	1.442	1.903
100	1.654	1.694	1.634	1.715	1.613	1.736	1.592	1.758	1.571	1.780	1.550	1.803	1.528	1.826	1.506	1.850	1.484	1.874	1.462	1.898
150	1.720	1.746	1.706	1.760	1.693	1.774	1.679	1.788	1.665	1.802	1.651	1.817	1.637	1.832	1.622	1.847	1.608	1.862	1.594	1.877
200	1.758	1.778	1.748	1.789	1.738	1.799	1.728	1.810	1.718	1.820	1.707	1.831	1.697	1.841	1.686	1.852	1.675	1.863	1.665	1.874

P.T.O.

	$k'=11$		$k'=12$		$k'=13$		$k'=14$		$k'=15$		$k'=16$		$k'=17$		$k'=18$		$k'=19$		$k'=20$	
n	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U
16	0.098	3.503	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
17	0.138	3.378	0.087	3.557	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
18	0.177	3.265	0.123	3.441	0.078	3.603	—	—	—	—	—	—	—	—	—	—	—	—	—	—
19	0.220	3.159	0.160	3.335	0.111	3.496	0.070	3.642	—	—	—	—	—	—	—	—	—	—	—	—
20	0.263	3.063	0.200	3.234	0.145	3.395	0.100	3.542	0.063	3.676	—	—	—	—	—	—	—	—	—	—
21	0.307	2.976	0.240	3.141	0.182	3.300	0.132	3.448	0.091	3.583	0.058	3.705	—	—	—	—	—	—	—	—
22	0.349	2.897	0.281	3.057	0.220	3.211	0.166	3.358	0.120	3.495	0.083	3.619	0.052	3.731	—	—	—	—	—	—
23	0.391	2.826	0.322	2.979	0.259	3.128	0.202	3.272	0.153	3.409	0.110	3.535	0.076	3.650	0.048	3.753	—	—	—	—
24	0.431	2.761	0.362	2.908	0.297	3.053	0.239	3.193	0.186	3.327	0.141	3.454	0.101	3.572	0.070	3.678	0.044	3.773	—	—
25	0.470	2.702	0.400	2.844	0.335	2.983	0.275	3.119	0.221	3.251	0.172	3.376	0.130	3.494	0.094	3.604	0.065	3.702	0.041	3.790
26	0.508	2.649	0.438	2.784	0.373	2.919	0.312	3.051	0.256	3.179	0.205	3.303	0.160	3.420	0.120	3.531	0.087	3.632	0.060	3.724
27	0.544	2.600	0.475	2.730	0.409	2.859	0.348	2.987	0.291	3.112	0.238	3.233	0.191	3.349	0.149	3.460	0.112	3.563	0.081	3.658
28	0.578	2.555	0.510	2.680	0.445	2.805	0.383	2.928	0.325	3.050	0.271	3.168	0.222	3.283	0.178	3.392	0.138	3.495	0.104	3.592
29	0.612	2.515	0.544	2.634	0.479	2.755	0.418	2.874	0.359	2.992	0.305	3.107	0.254	3.219	0.208	3.327	0.166	3.431	0.129	3.528
30	0.643	2.477	0.577	2.592	0.512	2.708	0.451	2.823	0.392	2.937	0.337	3.050	0.286	3.160	0.238	3.266	0.195	3.368	0.156	3.465
31	0.674	2.443	0.608	2.553	0.545	2.665	0.484	2.776	0.425	2.887	0.370	2.996	0.317	3.103	0.269	3.208	0.224	3.309	0.183	3.406
32	0.703	2.411	0.638	2.517	0.576	2.625	0.515	2.733	0.457	2.840	0.401	2.946	0.349	3.050	0.299	3.153	0.253	3.252	0.211	3.348
33	0.731	2.382	0.668	2.484	0.606	2.588	0.546	2.692	0.488	2.796	0.432	2.899	0.379	3.000	0.329	3.100	0.283	3.198	0.239	3.293
34	0.758	2.355	0.695	2.454	0.634	2.554	0.575	2.654	0.518	2.754	0.462	2.854	0.409	2.954	0.359	3.051	0.312	3.147	0.267	3.240
35	0.783	2.330	0.722	2.425	0.662	2.521	0.604	2.619	0.547	2.716	0.492	2.813	0.439	2.910	0.388	3.005	0.340	3.099	0.295	3.190
36	0.808	2.306	0.748	2.398	0.689	2.492	0.631	2.586	0.575	2.680	0.520	2.774	0.467	2.868	0.417	2.961	0.369	3.053	0.323	3.142
37	0.831	2.285	0.772	2.374	0.714	2.464	0.657	2.555	0.602	2.646	0.548	2.738	0.495	2.829	0.445	2.920	0.397	3.009	0.351	3.097
38	0.854	2.265	0.796	2.351	0.739	2.438	0.683	2.526	0.628	2.614	0.575	2.703	0.522	2.792	0.472	2.880	0.424	2.968	0.378	3.054
39	0.875	2.246	0.819	2.329	0.763	2.413	0.707	2.499	0.653	2.585	0.600	2.671	0.549	2.757	0.499	2.843	0.451	2.929	0.404	3.013
40	0.896	2.228	0.840	2.309	0.785	2.391	0.731	2.473	0.678	2.557	0.626	2.641	0.575	2.724	0.525	2.808	0.477	2.892	0.430	2.974
45	0.988	2.156	0.938	2.225	0.887	2.296	0.838	2.367	0.788	2.439	0.740	2.512	0.692	2.586	0.644	2.659	0.598	2.733	0.553	2.807
50	1.064	2.103	1.019	2.163	0.973	2.225	0.927	2.287	0.882	2.350	0.836	2.414	0.792	2.479	0.747	2.544	0.703	2.610	0.660	2.675
55	1.129	2.062	1.087	2.116	1.045	2.170	1.003	2.225	0.961	2.281	0.919	2.338	0.877	2.396	0.836	2.454	0.795	2.512	0.754	2.571
60	1.184	2.031	1.145	2.079	1.106	2.127	1.068	2.177	1.029	2.227	0.990	2.278	0.951	2.330	0.913	2.382	0.874	2.434	0.836	2.487
65	1.231	2.006	1.195	2.049	1.160	2.093	1.124	2.138	1.088	2.183	1.052	2.229	1.016	2.276	0.980	2.323	0.944	2.371	0.908	2.419
70	1.272	1.986	1.239	2.026	1.206	2.066	1.172	2.106	1.139	2.148	1.105	2.189	1.072	2.232	1.038	2.275	1.005	2.318	0.971	2.362
75	1.308	1.970	1.277	2.006	1.247	2.043	1.215	2.080	1.184	2.118	1.153	2.156	1.121	2.195	1.090	2.235	1.058	2.275	1.027	2.315
80	1.340	1.957	1.311	1.991	1.283	2.024	1.253	2.059	1.224	2.093	1.195	2.129	1.165	2.165	1.136	2.201	1.106	2.238	1.076	2.275
85	1.369	1.946	1.342	1.977	1.315	2.009	1.287	2.040	1.260	2.073	1.232	2.105	1.205	2.139	1.177	2.172	1.149	2.206	1.121	2.241
90	1.395	1.937	1.369	1.966	1.344	1.995	1.318	2.025	1.292	2.055	1.266	2.085	1.240	2.116	1.213	2.148	1.187	2.179	1.160	2.211
95	1.418	1.929	1.394	1.956	1.370	1.984	1.345	2.012	1.321	2.040	1.296	2.068	1.271	2.097	1.247	2.126	1.222	2.156	1.197	2.186
100	1.439	1.923	1.416	1.948	1.393	1.974	1.371	2.000	1.347	2.026	1.324	2.053	1.301	2.080	1.277	2.108	1.253	2.135	1.229	2.164
150	1.579	1.892	1.564	1.908	1.550	1.924	1.535	1.940	1.519	1.956	1.504	1.972	1.489	1.989	1.474	2.006	1.458	2.023	1.443	2.040
200	1.654	1.885	1.643	1.896	1.632	1.908	1.621	1.919	1.610	1.931	1.599	1.943	1.588	1.955	1.576	1.967	1.565	1.979	1.554	1.991

Note: n = number of observations, k' = number of explanatory variables excluding the constant term.

Source: This table is an extension of the original Durbin-Watson table and is reproduced from N. E. Savin and K. J. White, "The Durbin-Watson Test for Serial Correlation with Extreme Small Samples or Many Regressors," *Econometrica*, vol. 45, November 1977, pp. 1989–96 and as corrected by R. W. Farebrother, *Econometrica*, vol. 48, September 1980, p. 1554. Reprinted by permission of the Econometric Society.

Example E.1.

If $n = 40$ and $k' = 4$, $d_L = 1.285$ and $d_U = 1.721$. If a computed d value is less than 1.285, there is evidence of positive first-order serial correlation; if it is greater than 1.721, there is no evidence of positive first-order serial correlation; but if d lies between the lower and the upper limit, there is inconclusive evidence regarding the presence or absence of positive first-order serial correlation.

TABLE E-5b DURBIN-WATSON d STATISTIC: SIGNIFICANCE POINTS OF d_L AND d_U AT 0.01 LEVEL OF SIGNIFICANCE

	$k'=1$		$k'=2$		$k'=3$		$k'=4$		$k'=5$		$k'=6$		$k'=7$		$k'=8$		$k'=9$		$k'=10$	
n	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U
6	0.390	1.142	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
7	0.435	1.036	0.294	1.676	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
8	0.497	1.003	0.345	1.489	0.229	2.102	—	—	—	—	—	—	—	—	—	—	—	—	—	—
9	0.554	0.998	0.408	1.389	0.279	1.875	0.183	2.433	—	—	—	—	—	—	—	—	—	—	—	—
10	0.604	1.001	0.466	1.333	0.340	1.733	0.230	2.193	0.150	2.690	—	—	—	—	—	—	—	—	—	—
11	0.653	1.010	0.519	1.297	0.396	1.640	0.286	2.030	0.193	2.453	0.124	2.892	—	—	—	—	—	—	—	—
12	0.697	1.023	0.569	1.274	0.449	1.575	0.339	1.913	0.244	2.280	0.164	2.665	0.105	3.053	—	—	—	—	—	—
13	0.738	1.038	0.616	1.261	0.499	1.526	0.391	1.826	0.294	2.150	0.211	2.490	0.140	2.838	0.090	3.182	—	—	—	—
14	0.776	1.054	0.660	1.254	0.547	1.490	0.441	1.757	0.343	2.049	0.257	2.354	0.183	2.667	0.122	2.981	0.078	3.287	—	—
15	0.811	1.070	0.700	1.252	0.591	1.464	0.488	1.704	0.391	1.967	0.303	2.244	0.226	2.530	0.161	2.817	0.107	3.101	0.068	3.374
16	0.844	1.086	0.737	1.252	0.653	1.446	0.532	1.663	0.437	1.900	0.349	2.153	0.269	2.416	0.200	2.681	0.142	2.944	0.094	3.201
17	0.874	1.102	0.772	1.255	0.672	1.432	0.574	1.630	0.480	1.847	0.393	2.078	0.313	2.319	0.241	2.566	0.179	2.811	0.127	3.053
18	0.902	1.118	0.805	1.259	0.708	1.422	0.613	1.604	0.522	1.803	0.435	2.015	0.355	2.238	0.282	2.467	0.216	2.697	0.160	2.925
19	0.928	1.132	0.835	1.265	0.742	1.415	0.650	1.584	0.561	1.767	0.476	1.963	0.396	2.169	0.322	2.381	0.255	2.597	0.196	2.813
20	0.952	1.147	0.863	1.271	0.773	1.411	0.685	1.567	0.598	1.737	0.515	1.918	0.436	2.110	0.362	2.308	0.294	2.510	0.232	2.714
21	0.975	1.161	0.890	1.277	0.803	1.408	0.718	1.554	0.633	1.712	0.552	1.881	0.474	2.059	0.400	2.244	0.331	2.434	0.268	2.625
22	0.997	1.174	0.914	1.284	0.831	1.407	0.748	1.543	0.667	1.691	0.587	1.849	0.510	2.015	0.437	2.188	0.368	2.367	0.304	2.548
23	1.018	1.187	0.938	1.291	0.858	1.407	0.777	1.534	0.698	1.673	0.620	1.821	0.545	1.977	0.473	2.140	0.404	2.308	0.340	2.479
24	1.037	1.199	0.960	1.298	0.882	1.407	0.805	1.528	0.728	1.658	0.652	1.797	0.578	1.944	0.507	2.097	0.439	2.255	0.375	2.417
25	1.055	1.211	0.981	1.305	0.906	1.409	0.831	1.523	0.756	1.645	0.682	1.776	0.610	1.915	0.540	2.059	0.473	2.209	0.409	2.362
26	1.072	1.222	1.001	1.312	0.928	1.411	0.855	1.518	0.783	1.635	0.711	1.759	0.640	1.889	0.572	2.026	0.505	2.168	0.441	2.313
27	1.089	1.233	1.019	1.319	0.949	1.413	0.878	1.515	0.808	1.626	0.738	1.743	0.669	1.867	0.602	1.997	0.536	2.131	0.473	2.269
28	1.104	1.244	1.037	1.325	0.969	1.415	0.900	1.513	0.832	1.618	0.764	1.729	0.696	1.847	0.630	1.970	0.566	2.098	0.504	2.229
29	1.119	1.254	1.054	1.332	0.988	1.418	0.921	1.512	0.855	1.611	0.788	1.718	0.723	1.830	0.658	1.947	0.595	2.068	0.533	2.193
30	1.133	1.263	1.070	1.339	1.006	1.421	0.941	1.511	0.877	1.606	0.812	1.707	0.748	1.814	0.684	1.925	0.622	2.041	0.562	2.160
31	1.147	1.273	1.085	1.345	1.023	1.425	0.960	1.510	0.897	1.601	0.834	1.698	0.772	1.800	0.710	1.906	0.649	2.017	0.589	2.131
32	1.160	1.282	1.100	1.352	1.040	1.428	0.979	1.510	0.917	1.597	0.856	1.690	0.794	1.788	0.734	1.889	0.674	1.995	0.615	2.104
33	1.172	1.291	1.114	1.358	1.055	1.432	0.996	1.510	0.936	1.594	0.876	1.683	0.816	1.776	0.757	1.874	0.698	1.975	0.641	2.080
34	1.184	1.299	1.128	1.364	1.070	1.435	1.012	1.511	0.954	1.591	0.896	1.677	0.837	1.766	0.779	1.860	0.722	1.957	0.665	2.057
35	1.195	1.307	1.140	1.370	1.085	1.439	1.028	1.512	0.971	1.589	0.914	1.671	0.857	1.757	0.800	1.847	0.744	1.940	0.689	2.037
36	1.206	1.315	1.153	1.376	1.098	1.442	1.043	1.513	0.988	1.588	0.932	1.666	0.877	1.749	0.821	1.836	0.766	1.925	0.711	2.018
37	1.217	1.323	1.165	1.382	1.112	1.446	1.058	1.514	1.004	1.586	0.950	1.662	0.895	1.742	0.841	1.825	0.787	1.911	0.733	2.001
38	1.227	1.330	1.176	1.388	1.124	1.449	1.072	1.515	1.019	1.585	0.966	1.658	0.913	1.735	0.860	1.816	0.807	1.899	0.754	1.985
39	1.237	1.337	1.187	1.393	1.137	1.453	1.085	1.517	1.034	1.584	0.982	1.655	0.930	1.729	0.878	1.807	0.826	1.887	0.774	1.970
40	1.246	1.344	1.198	1.398	1.148	1.457	1.098	1.518	1.048	1.584	0.997	1.652	0.946	1.724	0.895	1.799	0.844	1.876	0.749	1.956
45	1.288	1.376	1.245	1.423	1.201	1.474	1.156	1.528	1.111	1.584	1.065	1.643	1.019	1.704	0.974	1.768	0.927	1.834	0.881	1.902
50	1.324	1.403	1.285	1.446	1.245	1.491	1.205	1.538	1.164	1.587	1.123	1.639	1.081	1.692	1.039	1.748	0.997	1.805	0.955	1.864
55	1.356	1.427	1.320	1.466	1.284	1.506	1.247	1.548	1.209	1.592	1.172	1.638	1.134	1.685	1.095	1.734	1.057	1.785	1.018	1.837
60	1.383	1.449	1.350	1.484	1.317	1.520	1.283	1.558	1.249	1.598	1.214	1.639	1.179	1.682	1.144	1.726	1.108	1.771	1.072	1.817
65	1.407	1.468	1.377	1.500	1.346	1.534	1.315	1.568	1.283	1.604	1.251	1.642	1.218	1.680	1.186	1.720	1.153	1.761	1.120	1.802
70	1.429	1.485	1.400	1.515	1.372	1.546	1.343	1.578	1.313	1.611	1.283	1.645	1.253	1.680	1.223	1.716	1.192	1.754	1.162	1.792
75	1.448	1.501	1.422	1.529	1.395	1.557	1.368	1.587	1.340	1.617	1.313	1.649	1.284	1.682	1.256	1.714	1.227	1.748	1.199	1.783
80	1.466	1.515	1.441	1.541	1.416	1.568	1.390	1.595	1.364	1.624	1.338	1.653	1.312	1.683	1.285	1.714	1.259	1.745	1.232	1.777
85	1.482	1.528	1.458	1.553	1.435	1.578	1.411	1.603	1.386	1.630	1.362	1.657	1.337	1.685	1.312	1.714	1.287	1.743	1.262	1.773
90	1.496	1.540	1.474	1.563	1.452	1.587	1.429	1.611	1.406	1.636	1.383	1.661	1.360	1.687	1.336	1.714	1.312	1.741	1.288	1.769
95	1.510	1.552	1.489	1.573	1.468	1.596	1.446	1.618	1.425	1.642	1.403	1.666	1.381	1.690	1.358	1.715	1.336	1.741	1.313	1.767
100	1.522	1.562	1.503	1.583	1.482	1.604	1.462	1.625	1.441	1.647	1.421	1.670	1.400	1.693	1.378	1.717	1.357	1.741	1.335	1.765
150	1.611	1.637	1.598	1.651	1.584	1.665	1.571	1.679	1.557	1.693	1.543	1.708	1.530	1.722	1.515	1.737	1.501	1.752	1.486	1.767
200	1.664	1.684	1.653	1.693	1.643	1.704	1.633	1.715	1.623	1.725	1.613	1.735	1.603	1.746	1.592	1.757	1.582	1.768	1.571	1.779

P.T.O.

	$k'=11$		$k'=12$		$k'=13$		$k'=14$		$k'=15$		$k'=16$		$k'=17$		$k'=18$		$k'=19$		$k'=20$	
n	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U
16	0.060	3.446	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
17	0.084	3.286	0.053	3.506	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
18	0.113	3.146	0.075	3.358	0.047	3.357	—	—	—	—	—	—	—	—	—	—	—	—	—	—
19	0.145	3.023	0.102	3.227	0.067	3.420	0.043	3.601	—	—	—	—	—	—	—	—	—	—	—	—
20	0.178	2.914	0.131	3.109	0.092	3.297	0.061	3.474	0.038	3.639	—	—	—	—	—	—	—	—	—	—
21	0.212	2.817	0.162	3.004	0.119	3.185	0.084	3.358	0.055	3.521	0.035	3.671	—	—	—	—	—	—	—	—
22	0.246	2.729	0.194	2.909	0.148	3.084	0.109	3.252	0.077	3.412	0.050	3.562	0.032	3.700	—	—	—	—	—	—
23	0.281	2.651	0.227	2.822	0.178	2.991	0.136	3.155	0.100	3.311	0.070	3.459	0.046	3.597	0.029	3.725	—	—	—	—
24	0.315	2.580	0.260	2.744	0.209	2.906	0.165	3.065	0.125	3.218	0.092	3.363	0.065	3.501	0.043	3.629	0.027	3.747	—	—
25	0.348	2.517	0.292	2.674	0.240	2.829	0.194	2.982	0.152	3.131	0.116	3.274	0.085	3.410	0.060	3.538	0.039	3.657	0.025	3.766
26	0.381	2.460	0.324	2.610	0.272	2.758	0.224	2.906	0.180	3.050	0.141	3.191	0.107	3.325	0.079	3.452	0.055	3.572	0.036	3.682
27	0.413	2.409	0.356	2.552	0.303	2.694	0.253	2.836	0.208	2.976	0.167	3.113	0.131	3.245	0.100	3.371	0.073	3.490	0.051	3.602
28	0.444	2.363	0.387	2.499	0.333	2.635	0.283	2.772	0.237	2.907	0.194	3.040	0.156	3.169	0.122	3.294	0.093	3.412	0.068	3.524
29	0.474	2.321	0.417	2.451	0.363	2.582	0.313	2.713	0.266	2.843	0.222	2.972	0.182	3.098	0.146	3.220	0.114	3.338	0.087	3.450
30	0.503	2.283	0.447	2.407	0.393	2.533	0.342	2.659	0.294	2.785	0.249	2.909	0.208	3.032	0.171	3.152	0.137	3.267	0.107	3.379
31	0.531	2.248	0.475	2.367	0.422	2.487	0.371	2.609	0.322	2.730	0.277	2.851	0.234	2.970	0.196	3.087	0.160	3.201	0.128	3.311
32	0.558	2.216	0.503	2.330	0.450	2.446	0.399	2.563	0.350	2.680	0.304	2.797	0.261	2.912	0.221	3.026	0.184	3.137	0.151	3.246
33	0.585	2.187	0.530	2.296	0.477	2.408	0.426	2.520	0.377	2.633	0.331	2.746	0.287	2.858	0.246	2.969	0.209	3.078	0.174	3.184
34	0.610	2.160	0.556	2.266	0.503	2.373	0.452	2.481	0.404	2.590	0.357	2.699	0.313	2.808	0.272	2.915	0.233	3.022	0.197	3.126
35	0.634	2.136	0.581	2.237	0.529	2.340	0.478	2.444	0.430	2.550	0.383	2.655	0.339	2.761	0.297	2.865	0.257	2.969	0.221	3.071
36	0.658	2.113	0.605	2.210	0.554	2.310	0.504	2.410	0.455	2.512	0.409	2.614	0.364	2.717	0.322	2.818	0.282	2.919	0.244	3.019
37	0.680	2.092	0.628	2.186	0.578	2.282	0.528	2.379	0.480	2.477	0.434	2.576	0.389	2.675	0.347	2.774	0.306	2.872	0.268	2.969
38	0.702	2.073	0.651	2.164	0.601	2.256	0.552	2.350	0.504	2.445	0.458	2.540	0.414	2.637	0.371	2.733	0.330	2.828	0.291	2.923
39	0.723	2.055	0.673	2.143	0.623	2.232	0.575	2.323	0.528	2.414	0.482	2.507	0.438	2.600	0.395	2.694	0.354	2.787	0.315	2.879
40	0.744	2.039	0.694	2.123	0.645	2.210	0.597	2.297	0.551	2.386	0.505	2.476	0.461	2.566	0.418	2.657	0.377	2.748	0.338	2.838
45	0.835	1.972	0.790	2.044	0.744	2.118	0.700	2.193	0.655	2.269	0.612	2.346	0.570	2.424	0.528	2.503	0.488	2.582	0.448	2.661
50	0.913	1.925	0.871	1.987	0.829	2.051	0.787	2.116	0.746	2.182	0.705	2.250	0.665	2.318	0.625	2.387	0.586	2.456	0.548	2.526
55	0.979	1.891	0.940	1.945	0.902	2.002	0.863	2.059	0.825	2.117	0.786	2.176	0.748	2.237	0.711	2.298	0.674	2.359	0.637	2.421
60	1.037	1.865	1.001	1.914	0.965	1.964	0.929	2.015	0.893	2.067	0.857	2.120	0.822	2.173	0.786	2.227	0.751	2.283	0.716	2.338
65	1.087	1.845	1.053	1.889	1.020	1.934	0.986	1.980	0.953	2.027	0.919	2.075	0.886	2.123	0.852	2.172	0.819	2.221	0.786	2.272
70	1.131	1.831	1.099	1.870	1.068	1.911	1.037	1.953	1.005	1.995	0.974	2.038	0.943	2.082	0.911	2.127	0.880	2.172	0.849	2.217
75	1.170	1.819	1.141	1.856	1.111	1.893	1.082	1.931	1.052	1.970	1.023	2.009	0.993	2.049	0.964	2.090	0.934	2.131	0.905	2.172
80	1.205	1.810	1.177	1.844	1.150	1.878	1.122	1.913	1.094	1.949	1.066	1.984	1.039	2.022	1.011	2.059	0.983	2.097	0.955	2.135
85	1.236	1.803	1.210	1.834	1.184	1.866	1.158	1.898	1.132	1.931	1.106	1.965	1.080	1.999	1.053	2.033	1.027	2.068	1.000	2.104
90	1.264	1.798	1.240	1.827	1.215	1.856	1.191	1.886	1.166	1.917	1.141	1.948	1.116	1.979	1.091	2.012	1.066	2.044	1.041	2.077
95	1.290	1.793	1.267	1.821	1.244	1.848	1.221	1.876	1.197	1.905	1.174	1.934	1.150	1.963	1.126	1.993	1.102	2.023	1.079	2.054
100	1.314	1.790	1.292	1.816	1.270	1.841	1.248	1.868	1.225	1.895	1.203	1.922	1.181	1.949	1.158	1.977	1.136	2.006	1.113	2.034
150	1.473	1.783	1.458	1.799	1.444	1.814	1.429	1.830	1.414	1.847	1.400	1.863	1.385	1.880	1.370	1.897	1.355	1.913	1.340	1.931
200	1.561	1.791	1.550	1.801	1.539	1.813	1.528	1.824	1.518	1.836	1.507	1.847	1.495	1.860	1.484	1.871	1.474	1.883	1.462	1.896

Note: n = number of observations, k' = number of explanatory variables excluding the constant term.

[This question paper contains 4 printed pages.]

Your Roll No............

Sr. No. of Question Paper : 5285 **H**

Unique Paper Code : 2273102006

Name of the Paper : MONEY AND FINANCIAL MARKETS

Name of the Course : **B.A. (HONS.) ECONOMICS**

Semester : IV

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions carry equal marks.
3. Answer any **Five** questions.

1. (a) The Third Working Group on Money Supply (1998) under the chairmanship of Dr YV Reddy observed that the prevailing set of monetary aggregates were not in conformity with the largely followed norms

of progressivity in terms of liquidity. Briefly discuss the major recommendations made by the group in order to address the problem with the prevailing set of monetary aggregates. (9)

(b) Suppose the required reserve ratio is 10% of deposits, banks hold 5% of deposits as excess reserves and the public holds 30% deposits as currency.

(i) Calculate the total money supply if monetary base is Rs. 30 lakh. (3)

(ii) If Central Bank engages in open market sale of Rs. 10 lakh, calculate the change in money supply. (3)

(iii) If the currency deposits ratio falls to 25%, what is the impact on exogenous money supply? (3)

2. (a) "The maximum loss from buying a call option is the option price and there is substantial upside potential for the buyer" – Explain this statement with a suitable example. (9)

(b) Briefly explain the major Banking Reforms in India since 1991. (9)

3. (a) A Coupon Bond maturing in one year has a face value of Rs. 1000 and coupon rate of 8%. If the market interest rate is 6% at the time of purchase, calculate :

(i) Price of the coupon bond (2)

(ii) Current yield (2)

(iii) Yield to maturity of this bond (2)

(b) Write short note on any **two** (6 Marks each)

(i) Difference between Future and Forward Contracts

(ii) Risk-return profile of Put Option (Long Put Position)

(iii) Differentiate between Exogenous and Endogenous money supply curves.

4. (a) 'It is not merely the number of instruments that is important, but the number of instruments exerting independent effects on the target variables'- Explain this statement in the context of monetary policy targets and achievements. (9)

(b) Discuss the effectiveness of M CLR system in improving the transmission effects of monetary policy. (9)

5. Explain how the expectations and segmented-markets hypothesis are extreme versions of the preferred-habitat hypothesis. Give an example to illustrate your explanation. (18)

6. (a) Discuss the implementation lag and effectiveness lag in monetary policy. (10)

(b) What are the key components of the monetary policy framework? Discuss the multiple indicator approach to monetary policy? (8)